

THE SUBGROUP LATTICE OF $Sz(32) : 5$

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Nr.	Structure	Order	Length	Maximal Subgroups	Minimal Overgroups
1	$Sz(32) : 5$	162688000	1	2, 3 (1025), 10 (198400), 12 (325376), 15 (524800)	
2	$Sz(32)$	32537600	1	4 (1025), 19 (198400), 24 (325376), 32 (524800)	1
3	$2^3 \times 4 \cdot 2 \times 2 \cdot 2 \cdot 2 \cdot 2 : 31 : 5$	158720	1025	4, 5 (31), 6 (32)	1
4	$2^3 \times 4 \cdot 2 \times 2 \cdot 2 \cdot 2 : 31$	31744	1025	8, 9 (32)	2, 3
5	$2^3 \times 4 \cdot 2 \times 2 \cdot 2 \cdot 2 : 5 \cdot 2$	5120	31775	7, 8, 14 (16)	3
6	$2^5 : 31 : 5$	4960	32800	9, 20 (31), 21 (32)	3
7	$2^3 \times 4 \cdot 2 \times 2 \cdot 2 \cdot 2 : 5$	2560	31775	11, 20 (16)	5
8	$2^3 \times 4 \cdot 2 \times 2 \cdot 2 \cdot 2 \cdot 2$	1024	1025	11 (31)	4, 5 (31)
9	$2^5 : 31$	992	32800	36, 42 (32)	4, 6
10	$41 : 5 : 2 \cdot 2$	820	198400	13, 19, 46 (41)	1
11	$2^3 \times 4 \cdot 2 \times 2 \cdot 2 \cdot 2$	512	31775	16 (15)	7, 8
12	$5^{1+2} : 4$	500	325376	17, 24, 25 (5)	1
13	$41 : 5 : 2$	410	198400	18, 26, 56 (41)	10
14	$2^4 : 5 \times 2 \cdot 2$	320	508400	20, 28, 46 (16)	5
15	$31 : 5 : 2$	310	524800	21, 32, 56 (31)	1
16	$2^3 \times 4 \cdot 2 \times 2 \cdot 2$	256	158875	22 (7)	11 (3)
17	$5^{1+2} : 2$	250	325376	23, 33, 34 (5)	12
18	$41 : 5$	205	198400	35, 62 (41)	13
19	$D_{82} \cdot 2$	164	198400	26, 64 (41)	2, 10
20	$2^4 : 5 \times 2$	160	508400	27, 36, 56 (16)	6 (2), 7, 14
21	$31 : 5$	155	524800	42, 62 (31)	6 (2), 15
22	$2^3 \times 4 \cdot 2 \times 2$	128	158875	29 (4), 30 (4), 31 (4), 28 (3)	16 (7)
23	5^{1+2}	125	325376	43, 44 (4), 45	17
24	$D_{50} \cdot 2$	100	325376	33, 47 (5)	2, 12
25	$5^2 : 2 \cdot 2$	100	1626880	34, 47, 46 (5)	12
26	D_{82}	82	198400	35, 66 (41)	13, 19

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Nr.	Structure	Order	Length	Maximal Subgroups	Minimal Overgroups
27	$2^4 : 5$	80	508400	48, 62 (16)	20
28	$2^4 \times 4$	64	31775	37 (10), 38 (10), 39 (10), 36	14 (16), 22 (15)
29	$2^3 \times 4 \cdot 2$	64	635500	40 (2), 41 (2), 37, 39 (2)	22
30	$2^3 \times 4 \cdot 2$	64	635500	41 (2), 40 (2), 39, 38, 37	22
31	$2^3 \times 4 \cdot 2$	64	635500	40 (2), 41 (2), 37, 38 (2)	22
32	D_{62}	62	524800	42, 66 (31)	2, 15
33	D_{50}	50	325376	45, 57 (5)	17, 24
34	$5^2 : 2$	50	1626880	43, 57, 56 (5)	17, 25
35	41	41	198400	67	18, 26
36	2^5	32	1025	48 (31)	9 (32), 20 (496), 28 (31)
37	$2^3 \times 4$	32	317750	49 (2), 50 (4), 51 (2), 52 (4), 53 (2), 48	28, 29 (2), 30 (2), 31 (2)
38	$2^3 \times 4$	32	317750	54 (4), 50 (2), 55 (2), 49 (2), 51 (4), 48	28, 30 (2), 31 (4)
39	$2^3 \times 4$	32	317750	55 (4), 52 (2), 54 (2), 53 (4), 49 (2), 48	28, 29 (4), 30 (2)
40	$2^2 \times 4 \cdot 2$	32	1271000	52, 54, 49	29, 30, 31
41	$2^2 \times 4 \cdot 2$	32	1271000	52, 54, 49	29, 30, 31
42	31	31	524800	67	9 (2), 21, 32
43	5^2	25	325376	62 (5), 63	23, 34 (5)
44	25	25	1301504	63	23
45	25	25	325376	63	23, 33
46	20	20	8134400	56, 64	10, 14, 25
47	$D_{10} \cdot 2$	20	1626880	57, 64 (5)	24, 25
48	2^4	16	31775	58 (15)	27 (16), 36, 37 (10), 38 (10), 39 (10)
49	$2^2 \times 4$	16	635500	59 (2), 60 (2), 61 (2), 58	37, 38, 39, 40 (2), 41 (2)
50	$2^2 \times 4$	16	635500	61 (2), 60 (4), 58	37 (2), 38
51	$2^2 \times 4$	16	635500	59 (2), 60 (4), 58	37, 38 (2)
52	$2^2 \times 4$	16	635500	61 (4), 60 (2), 58	37 (2), 39, 40 (2), 41 (2)
53	$2^2 \times 4$	16	635500	59 (2), 61 (4), 58	37, 39 (2)
54	$2^2 \times 4$	16	635500	59 (4), 60 (2), 58	38 (2), 39, 40 (2), 41 (2)
55	$2^2 \times 4$	16	635500	61 (2), 59 (4), 58	38, 39 (2)
56	10	10	8134400	62, 66	13, 15 (2), 20, 34, 46
57	D_{10}	10	1626880	63, 66 (5)	33, 34, 47

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Nr.	Structure	Order	Length	Maximal Subgroups	Minimal Overgroups
58	2^3	8	158875	65 (7)	48 (3), 49 (4), 50 (4), 51 (4), 52 (4), 53 (4), 54 (4), 55 (4)
59	2×4	8	1271000	64 (2), 65	49, 51, 53, 54 (2), 55 (2)
60	2×4	8	1271000	64 (2), 65	49, 50 (2), 51 (2), 52, 54
61	2×4	8	1271000	64 (2), 65	49, 50, 52 (2), 53 (2), 55
62	5	5	1626880	67	18 (5), 21 (10), 27 (5), 43, 56 (5)
63	5	5	325376	67	43, 44 (4), 45, 57 (5)
64	4	4	508400	66	19 (16), 46 (16), 47 (16), 59 (5), 60 (5), 61 (5)
65	2^2	4	158875	66 (3)	58 (7), 59 (8), 60 (8), 61 (8)
66	2	2	31775	67	26 (256), 32 (512), 56 (256), 57 (256), 64 (16), 65 (15)
67	1	1	1		35 (198400), 42 (524800), 62 (1626880), 63 (325376), 66 (31775)

TABLE 1. Subgroup lattice of $Sz(32) : 5$

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