

THE SUBGROUP LATTICE OF $PSp_4(3)$

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Nr.	Structure	Order	Length	Maximal Subgroups	Minimal Overgroups
1	$PSp_4(3)$	25920	1	2 (27), 3 (36), 4 (40), 5 (40), 6 (45)	
2	$2^4 : A_5$	960	27	12 (5), 15 (6), 22 (10), 31 (16)	1
3	$A_6 : 2$	720	36	7, 16 (6), 17 (6), 28 (10), 36 (15), 37 (15)	1
4	$3^{1+2} : (Q_8 : 3)$	648	40	10, 14 (4), 29 (9)	1
5	$3 : S_3 : S_3 : S_3$	648	40	8, 11 (3), 14 (4), 51 (27)	1
6	$2^{1+4} : 3 : S_3$	576	45	9, 13, 12 (3), 41 (16)	1
7	A_6	360	36	31 (6), 32 (6), 42 (10), 52 (15), 53 (15)	3
8	$3 : S_3 : S_3 : 3$	324	40	18, 26 (4), 76 (27)	5
9	$2^{1+4} : 3^2$	288	45	23, 24 (2), 25, 29 (8)	6
10	$3^{1+2} : Q_8$	216	40	19 (3), 54 (9)	4
11	$S_3 \times S_3 : S_3$	216	120	20, 18, 21, 28 (3), 55 (9)	5
12	$2^{1+4} : 6$	192	135	25, 30, 38 (4)	2, 6
13	$2^{1+4} : S_3$	192	45	23, 30 (3), 36 (12)	6
14	$3^3 : S_3$	162	160	26, 33, 34 (3)	4, 5
15	$2^4 : 5 : 2$	160	162	27, 45 (5), 83 (16)	2
16	S_5	120	216	31, 53 (5), 62 (6), 77 (10)	3
17	S_5	120	216	32, 51 (5), 62 (6), 78 (10)	3
18	$3 : S_3 : S_3$	108	40	35 (3), 43 (9)	8, 11 (3)
19	$3^{1+2} : 4$	108	120	33, 79 (9)	10
20	$S_3 \times 3 \times S_3$	108	120	34 (2), 35, 44, 41 (6)	11
21	$3 : S_3 \cdot S_3$	108	120	35, 42 (3), 80 (9)	11
22	$2^4 : S_3$	96	270	37, 38, 39, 45 (3), 55 (4)	2
23	$2^{1+4} : 3$	96	45	46, 56 (8), 57 (12)	9, 13
24	$2^{1+4} : 3$	96	90	40 (3), 46, 54 (4)	9
25	$2^{1+4} : 3$	96	45	46, 58 (8), 59 (12)	9, 12 (3)

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Nr.	Structure	Order	Length	Maximal Subgroups	Minimal Overgroups
26	$3^{1+2} : 3$	81	160	48, 49 (2), 50	8, 14
27	$2^4 : 5$	80	162	70, 104 (16)	15
28	$S_3 \times S_3 : 2$	72	360	42, 43, 44, 88 (9)	3, 11
29	$Q_8 : 3^2$	72	360	58, 56, 54, 60, 63 (4)	4, 9
30	$2^{1+4} : 2$	64	135	45 (3), 47 (3), 46	12, 13
31	A_5	60	216	81 (5), 83 (6), 97 (10)	2 (2), 7, 16
32	A_5	60	216	76 (5), 83 (6), 98 (10)	7, 17
33	$3^{1+2} : 2$	54	40	50, 64 (12)	14 (4), 19 (3)
34	$3^2 \times S_3$	54	240	48, 64 (2), 65, 66, 63 (3)	14 (2), 20
35	$3^2 : S_3$	54	120	48, 67, 68 (6), 69 (6)	18, 20, 21
36	$2^3 : S_3$	48	540	52, 51, 57, 71 (3), 78 (4)	3, 13
37	$2^3 : S_3$	48	270	53 (2), 61, 71 (3), 77 (4)	3 (2), 22
38	$A_4 \times 2 \times 2$	48	270	59 (2), 61, 70, 82 (4)	12 (2), 22
39	$2^3 \cdot S_3$	48	270	61, 72 (3), 80 (4)	22
40	$Q_8 \cdot 6$	48	270	60, 73, 79 (4)	24
41	$S_3 \times 6$	36	720	63, 65, 69, 78, 82 (3)	6, 20
42	$3 : S_3 \cdot 2$	36	360	67, 105 (9)	7, 21, 28
43	$S_3 \times S_3$	36	360	68 (2), 67, 77 (6)	18, 28
44	$S_3 \times S_3$	36	120	67, 66 (2), 78 (6)	20, 28 (3)
45	$D_8 \times 2 : 2$	32	405	70, 72 (2), 74, 75, 71 (2)	15 (2), 22 (2), 30
46	2^{1+4}	32	45	75 (9), 73 (6)	23, 24 (2), 25, 30 (3)
47	$D_8 \times 2 \cdot 2$	32	405	74 (2), 75	30
48	3^3	27	40	84 (4), 85 (3), 86 (6)	26 (4), 34 (6), 35 (3)
49	3^{1+2}	27	320	84, 87 (3)	26
50	3^{1+2}	27	40	84 (4)	26 (4), 33
51	$2^2 : S_3$	24	540	76, 88 (3), 99 (4)	5 (2), 17 (2), 36
52	$2^2 : S_3$	24	540	76, 89 (3), 98 (4)	7, 36
53	$2^2 : S_3$	24	540	81, 89 (3), 97 (4)	7, 16 (2), 37
54	$Q_8 \times 3$	24	360	79 (3), 90	10, 24, 29
55	$2^2 : S_3$	24	1080	80, 82, 77, 88 (3)	11, 22
56	$Q_8 : 3$	24	360	90, 100 (4)	23, 29
57	$A_4 \times 2$	24	540	76, 91, 100 (4)	23, 36
58	$Q_8 : 3$	24	360	90, 101 (4)	25, 29
59	$A_4 \times 2$	24	540	81, 92, 101 (4)	25, 38
60	$Q_8 : 3$	24	90	90, 102 (4)	29 (4), 40 (3)
61	$A_4 \times 2$	24	270	81, 93, 103 (4)	37, 38, 39
62	$D_{10} \cdot 2$	20	1296	83, 105 (5)	16, 17
63	3×6	18	720	86, 102 (2), 101, 100	29 (2), 34, 41
64	$S_3 \times 3$	18	480	84, 99, 102 (3)	33, 34

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65	$S_3 \times 3$	18	240	86, 99, 101 (3)	34, 41 (3)
66	$S_3 \times 3$	18	240	85, 99, 100 (3)	34, 44
67	$3 : S_3$	18	120	85, 97 (6), 98 (6)	35, 42 (3), 43 (3), 44
68	$S_3 \times 3$	18	720	85, 97, 103 (3)	35, 43
69	$S_3 \times 3$	18	720	86, 98, 103 (3)	35, 41
70	2^4	16	27	93 (10), 92 (5)	27 (6), 38 (10), 45 (15)
71	$D_8 \times 2$	16	810	93, 94, 88 (2), 89 (2), 91	36 (2), 37, 45
72	$2 \times 4 : 2$	16	810	93, 94, 95	39, 45
73	$D_8 : 2$	16	270	96 (3), 95 (3), 90	40, 46
74	$2 \times 4 : 2$	16	405	92, 94 (2)	45, 47 (2)
75	$D_8 \times 2$	16	405	96 (4), 95, 92, 91	45, 46, 47
76	A_4	12	540	106, 111 (4)	8 (2), 32 (2), 51, 52, 57
77	D_{12}	12	1080	103, 97 (2), 106 (3)	16 (2), 37, 43 (2), 55
78	D_{12}	12	720	98, 100, 99, 107 (3)	17 (3), 36 (3), 41, 44
79	12	12	1080	102, 108	19, 40, 54
80	Q_{12}	12	1080	103, 105 (3)	21, 39, 55
81	A_4	12	270	109, 112 (4)	31 (4), 53 (2), 59 (2), 61
82	2×6	12	1080	101 (2), 103, 107	38, 41 (2), 55
83	D_{10}	10	1296	104, 114 (5)	15 (2), 31, 32, 62
84	3^2	9	160	113, 111 (3)	48, 49 (2), 50, 64 (3)
85	3^2	9	120	112 (2), 111 (2)	48, 66 (2), 67, 68 (6)
86	3^2	9	240	113 (2), 112, 111	48, 63 (3), 65, 69 (3)
87	9	9	960	113	49
88	D_8	8	1620	107, 105, 106	28 (2), 51, 55 (2), 71
89	D_8	8	1620	106, 105, 109	52, 53, 71
90	Q_8	8	90	108 (3)	54 (4), 56 (4), 58 (4), 60, 73 (3)
91	2^3	8	135	110 (3), 106 (4)	57 (4), 71 (6), 75 (3)
92	2^3	8	135	110 (3), 109 (4)	59 (4), 70, 74 (3), 75 (3)
93	2^3	8	270	107 (3), 110 (3), 109	61, 70, 71 (3), 72 (3)
94	2×4	8	810	105 (2), 110	71, 72, 74
95	2×4	8	405	108 (2), 110	72 (2), 73 (2), 75
96	D_8	8	810	110 (2), 108	73, 75 (2)
97	S_3	6	720	112, 114 (3)	31 (3), 53 (3), 67, 68, 77 (3)
98	S_3	6	720	111, 114 (3)	32 (3), 52 (3), 67, 69, 78

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Nr.	Structure	Order	Length	Maximal Subgroups	Minimal Overgroups
99	S_3	6	240	111, 115 (3)	51 (9), 64 (2), 65, 66, 78 (3)
100	6	6	720	111, 115	56 (2), 57 (3), 63, 66, 78
101	6	6	720	112, 115	58 (2), 59 (3), 63, 65, 82 (3)
102	6	6	360	113, 115	60, 63 (4), 64 (4), 79 (3)
103	6	6	1080	112, 114	61, 68 (2), 69 (2), 77, 80, 82
104	5	5	1296	116	27 (2), 83
105	4	4	1620	114	42 (2), 62 (4), 80 (2), 88, 89, 94
106	2^2	4	540	114 (3)	76, 77 (6), 88 (3), 89 (3), 91
107	2^2	4	270	115 (2), 114	78 (8), 82 (4), 88 (6), 93 (3)
108	4	4	270	115	79 (4), 90, 95 (3), 96 (3)
109	2^2	4	270	114 (3)	81, 89 (6), 92 (2), 93
110	2^2	4	405	115, 114 (2)	91, 92, 93 (2), 94 (2), 95, 96 (4)
111	3	3	240	116	76 (9), 84 (2), 85, 86, 98 (3), 99, 100 (3)
112	3	3	120	116	81 (9), 85 (2), 86 (2), 97 (6), 101 (6), 103 (9)
113	3	3	40	116	84 (4), 86 (12), 87 (24), 102 (9)
114	2	2	270	116	83 (24), 97 (8), 98 (8), 103 (4), 105 (6), 106 (6), 107, 109 (3), 110 (3)
115	2	2	45	116	99 (16), 100 (16), 101 (16), 102 (8), 107 (12), 108 (6), 110 (9)
116	1	1	1		104 (1296), 111 (240), 112 (120), 113 (40), 114 (270), 115 (45)

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TABLE 1. Subgroup lattice of $PSp_4(3)$

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