

THE SUBGROUP LATTICE OF $U_3(5)$

THOMAS CONNOR AND DIMITRI LEEMANS

Nr.	Structure	Order	Length	Maximal Subgroups	Minimal Overgroups
1	$U_3(5)$	126000	1	2 (50), 3 (50), 4 (50), 5 (126), 6 (175), 7 (175), 8 (175), 14 (525)	
2	A_7	2520	50	10 (7), 15 (15), 16 (15), 19 (21), 26 (35)	1
3	A_7	2520	50	11 (7), 16 (15), 17 (15), 20 (21), 26 (35)	1
4	A_7	2520	50	12 (7), 15 (15), 17 (15), 21 (21), 26 (35)	1
5	$5^{1+2} : 8$	1000	126	9, 35 (25)	1
6	M_{10}	720	175	11, 27 (10), 49 (36), 54 (45)	1
7	M_{10}	720	175	10, 27 (10), 50 (36), 54 (45)	1
8	M_{10}	720	175	12, 27 (10), 51 (36), 54 (45)	1
9	$5^{1+2} : 4$	500	126	13, 23 (10), 24 (10), 25 (10)	5
10	A_6	360	175	28 (12), 36 (10), 43 (30)	2 (2), 7
11	A_6	360	175	29 (12), 37 (10), 44 (30)	3 (2), 6
12	A_6	360	175	30 (12), 38 (10), 45 (30)	4 (2), 8
13	$5^{1+2} : 2$	250	126	18, 31 (10), 32 (10), 33 (10)	9
14	$2 \cdot A_5 : 2$	240	525	22, 34 (5), 35 (6), 46 (10)	1
15	$L_2(7)$	168	750	45 (7), 43 (7), 48 (8)	2, 4
16	$L_2(7)$	168	750	43 (7), 44 (7), 48 (8)	2, 3
17	$L_2(7)$	168	750	44 (7), 45 (7), 48 (8)	3, 4
18	5^{1+2}	125	126	40 (2), 41 (2), 42 (2)	13
19	$A_5 : 2$	120	1050	28, 43 (5), 50 (6), 55 (10)	2

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20	$A_5 : 2$	120	1050	29, 44 (5), 49 (6), 55 (10)	3
21	$A_5 : 2$	120	1050	30, 45 (5), 51 (6), 55 (10)	4
22	$2 \cdot A_5$	120	525	47 (5), 52 (6), 56 (10)	14
23	$5^2 : 2 \cdot 2$	100	1260	31, 51 (5), 52 (5)	9
24	$5^2 : 2 \cdot 2$	100	1260	32, 52 (5), 50 (5)	9
25	$5^2 : 2 \cdot 2$	100	1260	33, 52 (5), 49 (5)	9
26	$A_4 : S_3$	72	1750	39, 44 (3), 45 (3), 46 (3), 43 (3), 53 (4)	2, 3, 4
27	M_9	72	1750	38, 36, 37, 66 (9)	6, 7, 8
28	A_5	60	1050	57 (5), 61 (6), 70 (10)	10 (2), 19
29	A_5	60	1050	58 (5), 62 (6), 70 (10)	11 (2), 20
30	A_5	60	1050	59 (5), 63 (6), 70 (10)	12 (2), 21
31	$5^2 : 2$	50	1260	41, 63, 64 (5)	13, 23
32	$5^2 : 2$	50	1260	42, 61, 64 (5)	13, 24
33	$5^2 : 2$	50	1260	40, 62, 64 (5)	13, 25
34	$Q_8 : S_3$	48	2625	47, 54 (3), 55 (4)	14
35	$Q_{20} \cdot 2$	40	3150	52, 67 (5)	5, 14
36	$3 : S_3 \cdot 2$	36	1750	53, 76 (9)	10, 27
37	$3 : S_3 \cdot 2$	36	1750	53, 76 (9)	11, 27
38	$3 : S_3 \cdot 2$	36	1750	53, 76 (9)	12, 27
39	$A_4 \times 3$	36	1750	59, 58, 60, 57, 65 (4)	26
40	5^2	25	252	72 (5), 73	18, 33 (5)
41	5^2	25	252	74 (5), 73	18, 31 (5)
42	5^2	25	252	75 (5), 73	18, 32 (5)
43	$2^2 : S_3$	24	5250	57, 68 (3), 70 (4)	10, 15, 16, 19, 26
44	$2^2 : S_3$	24	5250	58, 68 (3), 70 (4)	11, 16, 17, 20, 26
45	$2^2 : S_3$	24	5250	59, 68 (3), 70 (4)	12, 15, 17, 21, 26
46	$2^2 : S_3$	24	5250	56, 55, 60, 68 (3)	14, 26
47	$Q_8 : 3$	24	2625	66, 71 (4)	22, 34
48	$7 : 3$	21	6000	69, 78 (7)	15, 16, 17
49	$D_{10} \cdot 2$	20	6300	62, 76 (5)	6, 20, 25
50	$D_{10} \cdot 2$	20	6300	61, 76 (5)	7, 19, 24
51	$D_{10} \cdot 2$	20	6300	63, 76 (5)	8, 21, 23
52	Q_{20}	20	3150	64, 76 (5)	22, 23 (2), 24 (2), 25 (2), 35
53	$3 : S_3$	18	1750	65, 70 (12)	26 (4), 36, 37, 38
54	$Q_8 : 2$	16	7875	68, 67, 66	6, 7, 8, 34
55	D_{12}	12	5250	70 (2), 71, 77 (3)	19 (2), 20 (2), 21 (2), 34 (2), 46
56	Q_{12}	12	5250	71, 76 (3)	22, 46
57	A_4	12	1750	77, 78 (4)	28 (3), 39, 43 (3)

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Nr.	Structure	Order	Length	Maximal Subgroups	Minimal Overgroups
58	A_4	12	1750	77, 78 (4)	29 (3), 39, 44 (3)
59	A_4	12	1750	77, 78 (4)	30 (3), 39, 45 (3)
60	2×6	12	1750	71 (3), 77	39, 46 (3)
61	D_{10}	10	1260	75, 79 (5)	28 (5), 32, 50 (5)
62	D_{10}	10	1260	72, 79 (5)	29 (5), 33, 49 (5)
63	D_{10}	10	1260	74, 79 (5)	30 (5), 31, 51 (5)
64	10	10	3150	73, 79	31 (2), 32 (2), 33 (2), 52
65	3^2	9	1750	78 (4)	39 (4), 53
66	Q_8	8	2625	76 (3)	27 (6), 47, 54 (3)
67	8	8	7875	76	35 (2), 54
68	D_8	8	7875	77 (2), 76	43 (2), 44 (2), 45 (2), 46 (2), 54
69	7	7	6000	80	48
70	S_3	6	10500	78, 79 (3)	28, 29, 30, 43 (2), 44 (2), 45 (2), 53 (2), 55
71	6	6	5250	78, 79	47 (2), 55, 56, 60
72	5	5	1260	80	40, 62
73	5	5	126	80	40 (2), 41 (2), 42 (2), 64 (25)
74	5	5	1260	80	41, 63
75	5	5	1260	80	42, 61
76	4	4	7875	79	36 (2), 37 (2), 38 (2), 49 (4), 50 (4), 51 (4), 52 (2), 56 (2), 66, 67, 68
77	2^2	4	1750	79 (3)	55 (9), 57, 58, 59, 60, 68 (9)
78	3	3	1750	80	48 (24), 57 (4), 58 (4), 59 (4), 65 (4), 70 (6), 71 (3)
79	2	2	525	80	61 (12), 62 (12), 63 (12), 64 (6), 70 (60), 71 (10), 76 (15), 77 (10)
80	1	1	1		69 (6000), 72 (1260), 73 (126), 74 (1260), 75 (1260), 78 (1750), 79 (525)

TABLE 1. Subgroup lattice of $U_3(5)$

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