

THE SUBGROUP LATTICE OF $U_3(4) : 2$

THOMAS CONNOR AND DIMITRI LEEMANS

Nr.	Structure	Order	Length	Maximal Subgroups	Minimal Overgroups
1	$U_3(4) : 2$	124800	1	2, 3 (65), 6 (208), 9 (416), 20 (1600)	
2	$U_3(4)$	62400	1	4 (65), 10 (208), 12 (416), 33 (1600)	1
3	$2 \times 4 \cdot 2 \cdot 2 \cdot 2 : 5 : 3 : 2$	1920	65	4, 5, 7 (5), 16 (16)	1
4	$2 \times 4 \cdot 2 \cdot 2 \cdot 2 : 5 : 3$	960	65	8, 11, 25 (16)	2, 3
5	$2 \times 4 \cdot 2 \cdot 2 \cdot 2 : 5 : 2$	640	65	8, 15 (5), 32 (16)	3
6	$A_5 \times 5 : 2$	600	208	10, 17 (5), 16 (5), 18 (6), 26 (10)	1
7	$4^2 \cdot 2 \cdot 2 : 3 : 2$	384	325	11, 15, 19 (4)	3
8	$2 \times 4 \cdot 2 \cdot 2 \cdot 2 : 5$	320	65	22, 45 (16)	4, 5
9	$5^2 : 2 : S_3$	300	416	13, 14, 12, 18 (3), 56 (25)	1
10	$A_5 \times 5$	300	208	27, 25 (5), 28 (6), 39 (10)	2, 6
11	$2 \times 4 \cdot 2 \cdot 2 \cdot 2 : 3$	192	65	22, 31 (20)	4, 7 (5)
12	$5^2 : S_3$	150	416	21, 28 (3), 69 (25)	2, 9
13	$5^2 : 3 : 2$	150	416	21, 29, 70 (25)	9
14	$5^2 : S_3$	150	416	21, 30 (3), 71 (25)	9
15	$2 \times 4 \cdot 2 \cdot 2 \cdot 2 : 2$	128	325	23 (3), 24 (3), 22	5, 7
16	$2 \times 10 : 3 : 2$	120	1040	25, 32, 40 (4), 44 (5)	3, 6
17	$A_5 \times 2$	120	1040	27, 44 (5), 46 (6), 56 (10)	6
18	$5^2 : 2 : 2$	100	1248	30, 29, 28, 47 (5), 46 (5)	6, 9
19	$4^2 : 3 : 2$	96	1300	31, 34, 44 (4)	7
20	$13 : 3 : 2$	78	1600	33, 42, 70 (13)	1
21	$5^2 : 3$	75	416	43, 77 (25)	12, 13, 14
22	$2 \times 4 \cdot 2 \cdot 2 \cdot 2$	64	65	35 (15)	8, 11, 15 (5)
23	$D_8 : 4 : 2$	64	975	36 (2), 37 (2), 34, 38, 35	15
24	$2 \times 4 \cdot 2 \cdot 2 \cdot 2$	64	975	38 (2), 35	15
25	$2 \times 10 : 3$	60	1040	45, 54 (4), 57	4, 10, 16
26	$D_{10} \times S_3$	60	2080	40, 39, 41, 47 (3), 56 (5)	6

continued on next page

<i>continued from previous page</i>					
Nr.	Structure	Order	Length	Maximal Subgroups	Minimal Overgroups
27	A_5	60	208	57 (5), 58 (6), 69 (10)	10, 17 (5)
28	$5^2 : 2$	50	1248	43, 58, 59 (5)	10, 12, 18
29	$5^2 : 2$	50	416	43, 60 (15), 61 (15)	13, 18 (3)
30	$5^2 : 2$	50	1248	43, 62, 63 (5)	14, 18
31	$4^2 : 3$	48	1300	49, 57 (4)	11, 19
32	$D_{20} \times 2$	40	1040	48 (3), 47 (3), 45, 64 (5)	5, 16
33	$13 : 3$	39	1600	55, 77 (13)	2, 20
34	$D_8 \times 2 : 2$	32	325	50 (6), 49	19 (4), 23 (3)
35	$2 \times 4 \cdot 2 \cdot 2$	32	975	51 (3), 52 (3), 49	22, 23, 24
36	$D_8 : 4$	32	1950	50, 53, 52	23
37	$D_8 : 4$	32	1950	50, 53, 51	23
38	$2 \times 8 \cdot 2$	32	975	53 (2), 49	23, 24 (2)
39	$S_3 \times 5$	30	2080	54, 59 (3), 69	10, 26
40	$15 : 2$	30	2080	54, 62, 70 (5)	16 (2), 26
41	D_{30}	30	2080	54, 61 (3), 71 (5)	26
42	D_{26}	26	1600	55, 78 (13)	20
43	5^2	25	416	72 (3), 73 (3)	21, 28 (3), 29, 30 (3)
44	$A_4 \times 2$	24	5200	57, 64, 70 (4)	16, 17, 19
45	2×10	20	1040	59 (3), 74	8, 25, 32
46	D_{20}	20	6240	63, 58, 60, 75 (5)	17, 18
47	D_{20}	20	3120	59, 62, 61, 75 (5)	18 (2), 26 (2), 32
48	D_{20}	20	3120	61 (2), 59, 75 (5)	32
49	4^2	16	325	65 (3)	31 (4), 34, 35 (3), 38 (3)
50	$D_8 \times 2$	16	1950	66 (2), 67 (2), 64 (2), 65	34, 36, 37
51	$2 \times 4 \cdot 2$	16	975	65 (3)	35 (3), 37 (2)
52	$2 \times 4 \cdot 2$	16	975	65 (3)	35 (3), 36 (2)
53	2×8	16	1950	68 (2), 65	36, 37, 38
54	15	15	2080	73, 77	25 (2), 39, 40, 41
55	13	13	1600	80	33, 42
56	D_{12}	12	10400	69, 70, 71, 75 (3)	9, 17, 26
57	A_4	12	1040	74, 77 (4)	25, 27, 31 (5), 44 (5)
58	D_{10}	10	1248	72, 79 (5)	27, 28, 46 (5)
59	10	10	3120	73, 79	28 (2), 39 (2), 45, 47, 48
60	D_{10}	10	6240	72, 78 (5)	29, 46
61	D_{10}	10	3120	73, 78 (5)	29 (2), 41 (2), 47, 48 (2)
62	D_{10}	10	208	73, 78 (5)	30 (6), 40 (10), 47 (15)

continued on next page

<i>continued from previous page</i>					
Nr.	Structure	Order	Length	Maximal Subgroups	Minimal Overgroups
63	10	10	6240	72, 78	30, 46
64	2^3	8	1300	75 (6), 74	32 (4), 44 (4), 50 (3)
65	2×4	8	975	76 (2), 74	49, 50 (2), 51 (3), 52 (3), 53 (2)
66	D_8	8	3900	75 (2), 76	50
67	D_8	8	3900	75 (2), 76	50
68	8	8	3900	76	53
69	S_3	6	2080	77, 79 (3)	12 (5), 27, 39, 56 (5)
70	6	6	10400	77, 78	13, 20 (2), 40, 44 (2), 56
71	S_3	6	10400	77, 78 (3)	14, 41, 56
72	5	5	1248	80	43, 58, 60 (5), 63 (5)
73	5	5	208	80	43 (6), 54 (10), 59 (15), 61 (15), 62
74	2^2	4	65	79 (3)	45 (16), 57 (16), 64 (20), 65 (15)
75	2^2	4	7800	78 (2), 79	46 (4), 47 (2), 48 (2), 56 (4), 64, 66, 67
76	4	4	1950	79	65, 66 (2), 67 (2), 68 (2)
77	3	3	2080	80	21 (5), 33 (10), 54, 57 (2), 69, 70 (5), 71 (5)
78	2	2	1040	80	42 (20), 60 (30), 61 (15), 62, 63 (6), 70 (10), 71 (30), 75 (15)
79	2	2	195	80	58 (32), 59 (16), 69 (32), 74, 75 (40), 76 (10)
80	1	1	1		55 (1600), 72 (1248), 73 (208), 77 (2080), 78 (1040), 79 (195)

TABLE 1. Subgroup lattice of $U_3(4) : 2$

UNIVERSITÉ LIBRE DE BRUXELLES, DÉPARTEMENT DE MATHÉMATIQUES - C.P.216, BOULEVARD DU TRIOMPHE,
B-1050 BRUXELLES, BOURSIER FRIA

E-mail address: tconnor@ulb.ac.be

UNIVERSITY OF AUCKLAND, DEPARTMENT OF MATHEMATICS, PRIVATE BAG 92019, AUCKLAND, NEW ZEALAND

E-mail address: d.leemans@auckland.ac.nz