

# THE SUBGROUP LATTICE OF $L_3(5) : 2$

THOMAS CONNOR AND DIMITRI LEEMANS

Nr.	Structure	Order	Length	Maximal Subgroups	Minimal Overgroups
1	$L_3(5) : 2$	744000	1	2, 5 (186), 20 (775), 45 (3100), 55 (3875), 57 (4000)	
2	$L_3(5)$	372000	1	3 (62), 62 (3100), 74 (3875), 81 (4000)	1
3	$5^2 : (2 \cdot A_5) : 2 \cdot 2$	12000	62	4, 7 (5), 8 (6), 11 (10), 33 (25)	2
4	$5^2 : (2 \cdot A_5) : 2$	6000	62	6, 12 (5), 13 (6), 22 (10), 46 (25)	3
5	$5^{1+2} : (D_8 : 4)$	4000	186	9, 10, 8, 58 (25)	1
6	$5^2 : (2 \cdot A_5)$	3000	62	23 (5), 26 (6), 39 (10), 63 (25)	4
7	$5^2 : 2 \cdot 2 : 2 : 2 \cdot S_3$	2400	310	12, 21 (3), 24 (4), 75 (25)	3
8	$5^{1+2} : (4^2)$	2000	186	13 (2), 14, 36 (10)	3 (2), 5
9	$5^{1+2} : (8 : 2)$	2000	186	15 (2), 14, 82 (25)	5
10	$5^{1+2} : (D_8 : 2)$	2000	186	16, 17, 18 (2), 14, 19 (2), 83 (25)	5
11	$5^2 : 2 \cdot 2 \cdot 2 : S_3$	1200	620	24, 22, 25, 37 (3), 94 (25)	3
12	$5^2 : 2 \cdot 2 \cdot 2 : 3 : 2$	1200	310	23, 38, 40 (4), 95 (25)	4, 7
13	$5^{1+2} : (2 \times 4)$	1000	372	27, 28, 26, 49 (5), 50 (5)	4, 8
14	$5^{1+2} : (2 \times 4)$	1000	186	28, 29, 30, 49 (10)	8, 9, 10
15	$5^{1+2} : 8$	1000	372	30, 106 (25)	9
16	$5^{1+2} : D_8$	1000	186	31 (2), 29, 107 (25)	10
17	$5^{1+2} : Q_8$	1000	186	32 (2), 29, 108 (25)	10
18	$5^{1+2} : D_8$	1000	372	32, 31, 28, 109 (25)	10
19	$5^{1+2} : (2 \times 4)$	1000	372	31, 30, 32, 51 (10)	10
20	$Q_8 \cdot (A_5 : 2)$	960	775	34, 33, 35, 56 (5), 58 (6), 76 (10)	1
21	$5^2 : 2 \cdot 2 : 2 : 2 \cdot 2$	800	930	37, 38, 36, 114 (25)	7
22	$5^2 : 2 \cdot 2 : S_3$	600	620	40, 41, 39, 50 (3), 124 (25)	4, 11

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Nr.	Structure	Order	Length	Maximal Subgroups	Minimal Overgroups
23	$5^2 : 2 \cdot 2 \cdot 2 : 3$	600	310	52, 59 (4), 125 (25)	6, 12
24	$5^2 : 2 \cdot 2 \cdot S_3$	600	620	40, 53 (3), 126 (25)	7 (2), 11
25	$5^2 : 3 : 2 \cdot 2 \cdot 2$	600	620	40, 53, 127 (25)	11
26	$5^{1+2} : 4$	500	372	42, 66 (5), 67 (5)	6, 13
27	$5^{1+2} : 4$	500	372	42, 68, 69 (5)	13
28	$5^{1+2} : 2^2$	500	186	42 (2), 43, 70 (10)	13 (2), 14, 18 (2)
29	$5^{1+2} : 4$	500	186	43, 69 (10)	14, 16, 17
30	$5^{1+2} : 4$	500	186	43, 71 (20), 67 (10)	14, 15 (2), 19 (2)
31	$5^{1+2} : 2^2$	500	372	44 (2), 43, 72 (10)	16, 18, 19
32	$5^{1+2} : 4$	500	372	43, 73 (10)	17, 18, 19
33	$2 \cdot A_5 : 2 \cdot 2$	480	775	46, 75 (5), 84 (6), 94 (10)	3 (2), 20
34	$2 \cdot A_5 : 2 : 2$	480	775	47 (2), 46, 77 (5), 82 (6), 96 (10)	20
35	$Q_8 \cdot A_5$	480	775	48 (2), 46, 78 (5), 83 (6), 97 (10)	20
36	$5^2 : 2 : 2 \cdot 2 \cdot 2$	400	930	49 (2), 50, 84 (10)	8 (2), 21
37	$5^2 : 2 \cdot 2 \cdot 2 : 2$	400	930	53 (2), 50, 149 (25)	11 (2), 21
38	$5^2 : 2 \cdot 2 : 2 : 2$	400	310	54 (3), 50 (3), 52, 150 (25)	12, 21 (3)
39	$5^2 : 2 \cdot S_3$	300	620	59, 66 (3), 162 (25)	6, 22
40	$5^2 : 3 : 2 \cdot 2$	300	620	59, 68, 163 (25)	12 (2), 22, 24, 25
41	$5^2 : 2 : S_3$	300	620	60 (2), 59, 70 (3), 164 (25)	22
42	$5^{1+2} : 2$	250	372	61, 89, 90 (5)	26, 27, 28
43	$5^{1+2} : 2$	250	186	61, 90 (10), 91 (20)	28, 29, 30, 31 (2), 32 (2)
44	$5^{1+2} : 2$	250	744	61, 92, 93 (5)	31
45	$S_5 \times 2$	240	3100	64, 62, 65, 98 (5), 110 (6), 128 (10)	1
46	$2 \cdot A_5 : 2$	240	775	63, 95 (5), 111 (6), 124 (10)	4 (2), 33, 34, 35
47	$2 \cdot A_5 : 2$	240	1550	63, 99 (5), 106 (6), 129 (10)	34
48	$2 \cdot A_5 : 2$	240	1550	63, 100 (5), 112 (6), 130 (10)	35
49	$5^2 : 2 : 2 \cdot 2$	200	1860	69, 70, 67, 111 (5), 113 (5)	13, 14, 36
50	$5^2 : 2 : 2 \cdot 2$	200	930	70, 68, 66, 113 (10)	13 (2), 22 (2), 36, 37, 38
51	$5^2 : 2 \cdot 2 : 2$	200	3720	71, 73, 72, 110 (5), 112 (5)	19
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Nr.	Structure	Order	Length	Maximal Subgroups	Minimal Overgroups
52	$5^2 : 2 \cdot 2 \cdot 2$	200	310	66 (3), 178 (25)	23, 38
53	$5^2 : 2 \cdot 2 \cdot 2$	200	620	68, 179 (25)	24 (3), 25, 37 (3)
54	$5^2 : 2 \cdot 2 : 2$	200	930	70 (2), 66, 180 (25)	38
55	$D_8 \times 2 : 2 : S_3$	192	3875	74, 79, 80, 86 (3), 98 (4)	1
56	$2^{1+4} : S_3$	192	3875	78, 77, 75, 86 (3), 101 (4)	20
57	$31 : 3 : 2$	186	4000	81, 87, 192 (31)	1
58	$Q_8 : (D_{10} \cdot 2)$	160	4650	82, 83, 84, 115 (5)	5, 20
59	$5^2 : 3 : 2$	150	620	85, 89, 193 (25)	23 (2), 39, 40, 41
60	$5^2 : S_3$	150	1240	85, 90 (3), 194 (25)	41
61	$5^{1+2}$	125	186	122 (2), 123 (4)	42 (2), 43, 44 (4)
62	$A_5 : 2$	120	3100	88, 131 (5), 139 (6), 164 (10)	2, 45
63	$2 \cdot A_5$	120	775	125 (5), 140 (6), 162 (10)	6 (2), 46, 47 (2), 48 (2)
64	$A_5 : 2$	120	3100	88, 132 (5), 141 (6), 165 (10)	45
65	$A_5 \times 2$	120	3100	88, 133 (5), 142 (6), 165 (10)	45
66	$5^2 : 2 \cdot 2$	100	930	89, 143 (10)	26 (2), 39 (2), 50, 52, 54
67	$5^2 : 2 \cdot 2$	100	1860	90, 140 (5), 143 (5)	26, 30, 49
68	$5^2 : 2 \cdot 2$	100	62	89, 144 (30)	27 (6), 40 (10), 50 (15), 53 (10)
69	$5^2 : 2 \cdot 2$	100	1860	90, 144, 145 (5)	27, 29, 49
70	$5^2 : 2 : 2$	100	930	90 (2), 89, 146 (10)	28 (2), 41 (2), 49 (2), 50, 54 (2)
71	$5^2 : 2 \cdot 2$	100	3720	91, 139 (5), 140 (5)	30, 51
72	$5^2 : 2 : 2$	100	3720	92, 91, 93, 147 (5), 142 (5)	31, 51
73	$5^2 : 2 \cdot 2$	100	3720	91, 141, 148 (5)	32, 51
74	$4^2 : S_3$	96	3875	102, 114 (3), 131 (4)	2, 55
75	$D_8 : 2 \cdot S_3$	96	3875	95, 114 (3), 126 (4)	7 (2), 33, 56
76	$Q_8 : 2 : S_3$	96	7750	101, 103, 104, 96, 97, 105, 94, 116 (3)	20
77	$D_8 : 2 : S_3$	96	3875	99 (2), 95, 117 (3), 134 (4)	34, 56
78	$2^{1+4} : 3$	96	3875	95, 100 (2), 118, 135 (4)	35, 56
79	$4^2 : 3 : 2$	96	3875	102, 119, 133 (4)	55
80	$4^2 : S_3$	96	3875	102, 115 (3), 132 (4)	55
81	$31 : 3$	93	4000	121, 205 (31)	2, 57
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Nr.	Structure	Order	Length	Maximal Subgroups	Minimal Overgroups
82	$Q_{20} \cdot 2 : 2$	80	4650	106 (2), 111, 151 (5)	9, 34, 58
83	$Q_8 : (D_{10})$	80	4650	109 (2), 112 (2), 108, 107, 111, 152 (5)	10, 35, 58
84	$D_{20} \cdot 2 \cdot 2$	80	4650	113 (2), 111, 153 (5)	33, 36 (2), 58
85	$5^2 : 3$	75	620	122, 205 (25)	59, 60 (2)
86	$2^{1+4} : 2$	64	11625	119, 116, 115, 120, 114, 117, 118	55, 56
87	$D_{62}$	62	4000	121, 206 (31)	57
88	$A_5$	60	3100	166 (5), 172 (6), 194 (10)	62, 64, 65
89	$5^2 : 2$	50	62	122, 173 (30)	42 (6), 59 (10), 66 (15), 68, 70 (15)
90	$5^2 : 2$	50	1860	122, 173, 174 (5)	42, 43, 60 (2), 67, 69, 70
91	$5^2 : 2$	50	3720	123, 172, 174 (5)	43, 71, 72, 73
92	$5^2 : 2$	50	744	123, 175 (25), 176 (5)	44, 72 (5)
93	$5^2 : 2$	50	3720	123, 176, 177 (5)	44, 72
94	$8 : S_3$	48	7750	127, 126, 124, 149 (3)	11 (2), 33, 76
95	$Q_8 \cdot 6$	48	3875	125, 150, 163 (4)	12 (2), 46, 75, 77, 78
96	$D_8 \times S_3$	48	7750	136, 129 (2), 134, 128 (2), 124, 154 (3)	34, 76
97	$Q_8 : S_3$	48	7750	137 (2), 130 (2), 138, 135, 124, 152 (3)	35, 76
98	$2^3 : S_3$	48	15500	132, 133, 131, 155 (3), 167 (4)	45, 55
99	$Q_8 : S_3$	48	7750	125, 156 (3), 168 (4)	47, 77
100	$Q_8 \cdot 6$	48	7750	125, 157, 169 (4)	48, 78
101	$Q_8 : S_3$	48	7750	126, 135, 134, 158 (3)	56 (2), 76
102	$4^2 : 3$	48	3875	153, 166 (4)	74, 79, 80
103	$D_{48}$	48	7750	127, 134, 138, 159 (3)	76
104	$D_8 : S_3$	48	7750	136, 126, 138, 159 (3)	76
105	$Q_8 : 6$	48	7750	127, 135, 136, 158	76
106	$Q_{20} \cdot 2$	40	9300	140, 181 (5)	15, 47, 82
107	$D_{40}$	40	4650	147 (2), 145, 182 (5)	16, 83
108	$Q_8 \times 5$	40	4650	148 (2), 145, 183	17, 83
109	$D_{40}$	40	9300	148, 146, 147, 184 (5)	18, 83
110	$D_{10} \cdot 2 \times 2$	40	18600	139, 142, 141, 185 (5)	45, 51
111	$D_{20} \cdot 2$	40	4650	140, 145, 146, 186 (5)	46, 49 (2), 82, 83, 84
112	$D_{20} \cdot 2$	40	9300	140, 148, 147, 185 (5)	48, 51 (2), 83
113	$D_{10} \cdot 2 \times 2$	40	9300	143, 146, 144, 186 (5)	49, 50, 84
114	$Q_8 : 4$	32	11625	153, 149, 150	21 (2), 74, 75, 86
115	$D_8 : 4$	32	11625	153, 151, 152	58 (2), 80, 86
116	$Q_8 : 2^2$	32	11625	154, 149, 159 (2), 158 (2), 152	76 (2), 86
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Nr.	Structure	Order	Length	Maximal Subgroups	Minimal Overgroups
117	$Q_8 : 2^2$	32	11625	154, 151, 160 (2), 156 (2), 150	77, 86
118	$2^{1+4}$	32	3875	155 (6), 152 (3), 161 (3), 157 (2), 150	78, 86 (3)
119	$D_8 \times 2 : 2$	32	3875	161 (3), 154 (3), 153	79, 86 (3)
120	$8 : 2 : 2$	32	11625	161, 151, 149	86
121	31	31	4000	208	81, 87
122	$5^2$	25	62	197 (6)	61 (6), 85 (10), 89, 90 (30)
123	$5^2$	25	744	198 (5), 197	61, 91 (5), 92, 93 (5)
124	$S_3 \times 4$	24	7750	162, 163, 164, 186 (3)	22 (2), 46, 94, 96, 97
125	$Q_8 : 3$	24	3875	178, 193 (4)	23 (2), 63, 95, 99 (2), 100 (2)
126	$4 \cdot S_3$	24	7750	163, 179 (3)	24 (2), 75 (2), 94, 101, 104
127	24	24	7750	163, 179	25 (2), 94, 103, 105
128	$S_3 \times 2^2$	24	15500	165 (2), 168, 164, 167 (2), 170, 187 (3)	45 (2), 96
129	$2^2 : S_3$	24	15500	162, 170, 168, 188 (3)	47, 96
130	$S_3 \times 4$	24	15500	169, 162, 171, 185 (3)	48, 97
131	$2^2 : S_3$	24	15500	166, 180 (3), 194 (4)	62, 74, 98
132	$2^2 : S_3$	24	15500	166, 184 (3), 195 (4)	64, 80, 98
133	$A_4 \times 2$	24	15500	166, 189, 192 (4)	65, 79, 98
134	$D_{24}$	24	7750	163, 168 (2), 190 (3)	77 (2), 96, 101, 103
135	$Q_8 \times 3$	24	7750	169 (2), 163, 183	78 (2), 97, 101, 105
136	$D_8 \times 3$	24	7750	163, 170 (2), 190	96, 104, 105
137	$D_{24}$	24	15500	164, 169, 171, 184 (3)	97
138	$D_{24}$	24	7750	163, 171 (2), 182 (3)	97, 103, 104
139	$D_{10} \cdot 2$	20	18600	172, 199 (5)	62, 71, 110
140	$Q_{20}$	20	4650	174, 199 (5)	63, 67 (2), 71 (4), 106 (2), 111, 112 (2)
141	$D_{10} \cdot 2$	20	3720	172, 200 (5)	64 (5), 73, 110 (5)
142	$D_{20}$	20	18600	177, 172, 175, 201 (5)	65, 72, 110
143	$D_{10} \cdot 2$	20	9300	173, 199 (5)	66, 67, 113
144	$D_{10} \cdot 2$	20	1860	173, 202 (5)	68, 69, 113 (5)
145	20	20	4650	174, 202	69 (2), 107, 108, 111
146	$D_{20}$	20	4650	174, 173 (2), 203 (5)	70 (2), 109 (2), 111, 113 (2)
147	$D_{20}$	20	9300	176 (2), 174, 201 (5)	72 (2), 107, 109, 112
148	20	20	9300	174, 200	73 (2), 108, 109, 112
149	$8 : 2$	16	11625	186, 179 (2)	37 (2), 94 (2), 114, 116, 120
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Nr.	Structure	Order	Length	Maximal Subgroups	Minimal Overgroups
150	$D_8 : 2$	16	3875	180 (3), 178, 186 (3)	38 (2), 95, 114 (3), 117 (3), 118
151	$8 : 2$	16	11625	186, 181 (2)	82 (2), 115, 117, 120
152	$D_8 : 2$	16	11625	184 (2), 185 (2), 186, 183, 182	83 (2), 97 (2), 115, 116, 118
153	$4^2$	16	3875	186 (3)	84 (6), 102, 114 (3), 115 (3), 119
154	$D_8 \times 2$	16	11625	187 (2), 186, 190 (2), 188 (2)	96 (2), 116, 117, 119
155	$D_8 \times 2$	16	23250	184 (2), 185, 187, 191, 180, 189	98 (2), 118
156	$Q_8 : 2$	16	23250	188, 181, 178	99, 117
157	$D_8 : 2$	16	7750	191 (3), 185 (3), 178	100, 118
158	$Q_8 : 2$	16	7750	190, 179, 183	101 (3), 105, 116 (3)
159	$D_{16}$	16	23250	182, 179, 190	103, 104, 116
160	$D_{16}$	16	23250	180, 181, 188	117
161	$D_8 \times 2$	16	11625	191 (2), 182 (2), 187, 186, 189	118, 119, 120
162	$Q_{12}$	12	7750	193, 199 (3)	39 (2), 63, 124, 129 (2), 130 (2)
163	12	12	7750	193, 202	40 (2), 95 (2), 124, 126, 127, 134, 135, 136, 138
164	$D_{12}$	12	7750	194 (2), 193, 203 (3)	41 (2), 62 (4), 124, 128 (2), 137 (2)
165	$D_{12}$	12	31000	192, 195, 194, 201 (3)	64, 65, 128
166	$A_4$	12	15500	203, 205 (4)	88, 102, 131, 132, 133
167	$D_{12}$	12	31000	195, 192, 194, 204 (3)	98 (2), 128
168	$D_{12}$	12	15500	195 (2), 193, 204 (3)	99 (2), 128, 129, 134
169	12	12	15500	193, 200	100 (2), 130, 135, 137
170	$2 \times 6$	12	15500	193, 192 (2), 204	128, 129, 136
171	$D_{12}$	12	15500	196 (2), 193, 201 (3)	130, 137, 138
172	$D_{10}$	10	3720	198, 207 (5)	88 (5), 91, 139 (5), 141, 142 (5)
173	$D_{10}$	10	1860	197, 207 (5)	89, 90, 143 (5), 144, 146 (5)
174	10	10	4650	197, 207	90 (2), 91 (4), 140, 145, 146, 147 (2), 148 (2)
175	$D_{10}$	10	18600	198, 206 (5)	92, 142
176	$D_{10}$	10	3720	197, 206 (5)	92, 93, 147 (5)
177	10	10	18600	198, 206	93, 142
178	$Q_8$	8	3875	199 (3)	52 (2), 125, 150, 156 (6), 157 (2)
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Nr.	Structure	Order	Length	Maximal Subgroups	Minimal Overgroups
179	8	8	7750	202	53 (2), 126 (3), 127, 149 (3), 158, 159 (3)
180	$D_8$	8	11625	203 (2), 199	54 (2), 131 (4), 150, 155 (2), 160 (2)
181	8	8	23250	199	106 (2), 151, 156, 160
182	$D_8$	8	11625	201 (2), 202	107 (2), 138 (2), 152, 159 (2), 161 (2)
183	$Q_8$	8	775	202, 200 (2)	108 (6), 135 (10), 152 (15), 158 (10)
184	$D_8$	8	23250	201, 200, 203	109 (2), 132 (2), 137 (2), 152, 155 (2)
185	$2 \times 4$	8	23250	200, 199, 201	110 (4), 112 (2), 130 (2), 152, 155, 157
186	$2 \times 4$	8	11625	203, 199, 202	111 (2), 113 (4), 124 (2), 149, 150, 151, 152, 153, 154, 161
187	$2^3$	8	11625	204 (4), 201 (2), 203	128 (4), 154 (2), 155 (2), 161
188	$D_8$	8	23250	204 (2), 199	129 (2), 154, 156, 160
189	$2^3$	8	3875	201 (6), 203	133 (4), 155 (6), 161 (3)
190	$D_8$	8	7750	204 (2), 202	134 (3), 136, 154 (3), 158, 159 (3)
191	$D_8$	8	23250	201 (2), 199	155, 157, 161
192	6	6	31000	205, 206	57 (4), 133 (2), 165, 167, 170
193	6	6	7750	205, 207	59 (2), 125 (2), 162, 163, 164, 168 (2), 169 (2), 170 (2), 171 (2)
194	$S_3$	6	15500	205, 207 (3)	60 (2), 88 (2), 131 (4), 164, 165 (2), 167 (2)
195	$S_3$	6	31000	205, 206 (3)	132 (2), 165, 167, 168
196	$S_3$	6	31000	205, 206 (3)	171
197	5	5	186	208	122 (2), 123 (4), 173 (10), 174 (25), 176 (20)
198	5	5	3720	208	123, 172, 175 (5), 177 (5)
199	4	4	11625	207	139 (8), 140 (2), 143 (4), 162 (2), 178, 180, 181 (2), 185 (2), 186, 188 (2), 191 (2)
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Nr.	Structure	Order	Length	Maximal Subgroups	Minimal Overgroups
200	4	4	1550	207	141 (12), 148 (6), 169 (10), 183, 184 (15), 185 (15)
201	$2^2$	4	23250	206 (2), 207	142 (4), 147 (2), 165 (4), 171 (2), 182, 184, 185, 187, 189, 191 (2)
202	4	4	775	207	144 (12), 145 (6), 163 (10), 179 (10), 182 (15), 183, 186 (15), 190 (10)
203	$2^2$	4	3875	207 (3)	146 (6), 164 (6), 166 (4), 180 (6), 184 (6), 186 (3), 187 (3), 189
204	$2^2$	4	15500	207, 206 (2)	167 (6), 168 (3), 170, 187 (3), 188 (3), 190
205	3	3	7750	208	81 (16), 85 (2), 166 (8), 192 (4), 193, 194 (2), 195 (4), 196 (4)
206	2	2	3100	208	87 (40), 175 (30), 176 (6), 177 (6), 192 (10), 195 (30), 196 (30), 201 (15), 204 (10)
207	2	2	775	208	172 (24), 173 (12), 174 (6), 193 (10), 194 (60), 199 (15), 200 (2), 201 (30), 202, 203 (15), 204 (20)
208	1	1	1		121 (4000), 197 (186), 198 (3720), 205 (7750), 206 (3100), 207 (775)

TABLE 1. Subgroup lattice of  $L_3(5) : 2$ 

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