

THE SUBGROUP LATTICE OF $L_3(4) : S_3$

THOMAS CONNOR AND DIMITRI LEEMANS

| Nr. | Structure | Order | Length | Maximal Subgroups | Minimal Overgroups |
|-----|------------------------------------|--------|--------|---|--------------------|
| 1 | $L_3(4) : S_3$ | 120960 | 1 | 2, 3 (3), 6 (105), 11 (280), 14 (336), 29 (960) | |
| 2 | $L_3(4) : 3$ | 60480 | 1 | 4, 5 (42), 19 (280), 44 (960) | 1 |
| 3 | $L_3(4) : 2$ | 40320 | 3 | 4, 8 (56), 12 (105), 16 (120), 26 (280), 30 (336) | 1 |
| 4 | $L_3(4)$ | 20160 | 1 | 7 (42), 15 (168), 24 (360), 37 (280) | 2, 3 (3) |
| 5 | $2^4 : A_5 : 3$ | 2880 | 42 | 7, 9 (5), 10 (6), 17 (10), 23 (16) | 2 |
| 6 | $2 \times 4 : 2 : 2 : 2 : 3 : S_3$ | 1152 | 105 | 9, 13, 12 (3), 38 (16) | 1 |
| 7 | $2^4 : A_5$ | 960 | 42 | 20 (5), 25 (6), 32 (10), 45 (48), 46 (16) | 4, 5 |
| 8 | $A_6 : 2$ | 720 | 168 | 15, 39 (10), 82 (36), 89 (45) | 3 |
| 9 | $2 \times 4 : 2 : 2 : 2 : 3 : 3$ | 576 | 105 | 21 (2), 22, 20, 27 (8) | 5 (2), 6 |
| 10 | $2^4 : 5 : 3 : 2$ | 480 | 252 | 18, 25, 33 (5), 70 (16) | 5 |
| 11 | $M_9 : S_3$ | 432 | 280 | 19, 26 (3), 31 (4), 51 (9) | 1 |
| 12 | $2 \times 4 : 2 : 2 : 2 : 3 : 2$ | 384 | 315 | 20, 28, 34 (4) | 3, 6 |
| 13 | $Q_8 \times 2 : 2 : 2 : S_3$ | 384 | 105 | 22, 28 (3), 35 (12) | 6 |
| 14 | $A_5 \times S_3$ | 360 | 336 | 23, 30 (3), 38 (5), 47 (6), 60 (10) | 1 |
| 15 | A_6 | 360 | 168 | 45 (12), 61 (10), 74 (30) | 4, 8 |
| 16 | $L_2(7) : 2$ | 336 | 360 | 24, 58 (8), 90 (21), 102 (28) | 3 |
| 17 | $2^4 : 3 : S_3$ | 288 | 420 | 27, 32, 33 (3), 83 (16) | 5 |
| 18 | $2^4 : 5 : 3$ | 240 | 252 | 36, 52, 100 (16) | 10 |
| 19 | $M_9 : 3$ | 216 | 280 | 37, 48 (4), 75 (9) | 2, 11 |
| 20 | $2 \times 4 : 2 : 2 : 2 : 3$ | 192 | 105 | 41, 53 (12), 54 (8) | 7 (2), 9, 12 (3) |

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|-------------------------------------|------------------------------|-------|--------|------------------------------|--------------------------|
| Nr. | Structure | Order | Length | Maximal Subgroups | Minimal Overgroups |
| 21 | $2^4 : 3 : 2 : 2$ | 192 | 210 | 33 (3), 41, 55 (4) | 9 |
| 22 | $2 \times 4 : 2 : 2 : 2 : 3$ | 192 | 105 | 41, 56 (12), 55 (8) | 9, 13 |
| 23 | $A_5 \times 3$ | 180 | 336 | 46, 62 (5), 70 (6), 83 (10) | 5 (2), 14 |
| 24 | $L_2(7)$ | 168 | 360 | 74 (14), 79 (8) | 4, 16 |
| 25 | $2^4 : 5 : 2$ | 160 | 252 | 36, 64 (5), 109 (16) | 7, 10 |
| 26 | $M_9 : 2$ | 144 | 840 | 39, 37, 40, 91 (9) | 3, 11 |
| 27 | $A_4 \times 2 \times 2 : 3$ | 144 | 420 | 55 (2), 52, 54, 62 (8) | 9 (2), 17 |
| 28 | $2 \times 8 \cdot 2 : 2 : 2$ | 128 | 315 | 41, 42 (3), 43 (3) | 12, 13 |
| 29 | $7 : 3 : S_3$ | 126 | 960 | 44, 59, 58 (3), 84 (7) | 1 |
| 30 | $A_5 \times 2$ | 120 | 1008 | 46, 76 (5), 82 (6), 102 (10) | 3, 14 |
| 31 | $3^{1+2} : 2^2$ | 108 | 1120 | 48, 49, 50, 63 (3), 60 (3) | 11 |
| 32 | $2^4 : S_3$ | 96 | 420 | 54, 64 (3), 74 (12) | 7, 17 |
| 33 | $2^4 : 3 : 2$ | 96 | 630 | 52, 64, 77 (4) | 10 (2), 17 (2), 21 |
| 34 | $4^2 : 3 : 2$ | 96 | 1260 | 53, 65, 76 (4) | 12 |
| 35 | $Q_8 \times 2 : S_3$ | 96 | 1260 | 57, 51, 56, 66 (3), 78 (4) | 13 |
| 36 | $2^4 : 5$ | 80 | 252 | 92, 132 (16) | 18, 25 |
| 37 | M_9 | 72 | 280 | 61 (3), 115 (9) | 4, 19, 26 (3) |
| 38 | $A_4 \times S_3$ | 72 | 1680 | 62, 78, 76 (3), 85 (4) | 6, 14 |
| 39 | $3 : S_3 \cdot 2 \cdot 2$ | 72 | 840 | 61, 116 (9) | 8 (2), 26 |
| 40 | $3 : S_3 \cdot 2 : 2$ | 72 | 840 | 63 (2), 61, 117 (9) | 26 |
| 41 | $2 \times 4 : 2 : 2 : 2$ | 64 | 105 | 64 (6), 67 (9) | 20, 21 (2), 22, 28 (3) |
| 42 | $2 \times 8 \cdot 2 : 2$ | 64 | 945 | 67, 68 (2) | 28 |
| 43 | $D_8 \cdot 2^2 : 2$ | 64 | 945 | 69 (2), 66 (2), 67, 68, 65 | 28 |
| 44 | $7 : 3 \times 3$ | 63 | 960 | 80 (2), 81, 79, 112 (7) | 2, 29 |
| 45 | A_5 | 60 | 2016 | 103 (5), 109 (6), 125 (10) | 7, 15 |
| 46 | A_5 | 60 | 336 | 104 (5), 109 (6), 125 (10) | 7 (2), 23, 30 (3) |
| 47 | $D_{10} \times S_3$ | 60 | 2016 | 71, 72, 70, 82 (3), 105 (5) | 14 |
| 48 | $3^{1+2} : 2$ | 54 | 1120 | 73, 86, 83 (3) | 19, 31 |
| 49 | $3^{1+2} : 2$ | 54 | 1120 | 73, 85 (3), 84 (6), 87 (3) | 31 |
| 50 | $3^{1+2} : 2$ | 54 | 1120 | 73, 88, 87 (3) | 31 |
| 51 | $Q_8 : S_3$ | 48 | 1260 | 75, 91 (3), 105 (4) | 11 (2), 35 |
| 52 | $2^4 : 3$ | 48 | 42 | 92, 106 (20) | 18 (6), 27 (10), 33 (15) |

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| Nr. | Structure | Order | Length | Maximal Subgroups | Minimal Overgroups |
| 53 | $4^2 : 3$ | 48 | 1260 | 93, 104 (4) | 20, 34 |
| 54 | $2^4 : 3$ | 48 | 420 | 92, 103 (12), 104 (8) | 20 (2), 27, 32 |
| 55 | $A_4 \times 2 \times 2$ | 48 | 840 | 77 (3), 92, 107 (4) | 21, 22, 27 |
| 56 | $Q_8 : 6$ | 48 | 1260 | 75, 94, 107 (4) | 22, 35 |
| 57 | $Q_8 : S_3$ | 48 | 1260 | 75, 95 (3), 108 (4) | 35 |
| 58 | $7 : 3 : 2$ | 42 | 2880 | 79, 101, 126 (7) | 16, 29 |
| 59 | D_{42} | 42 | 960 | 81, 101 (3), 127 (7) | 29 |
| 60 | $S_3 \times S_3$ | 36 | 3360 | 85, 88, 83, 105 (3), 102 (3) | 14, 31 |
| 61 | $3 : S_3 \cdot 2$ | 36 | 840 | 86, 133 (9) | 15 (2), 37, 39, 40 |
| 62 | $A_4 \times 3$ | 36 | 1680 | 106 (2), 104, 107, 113 (4) | 23, 27 (2), 38 |
| 63 | $S_3 \times S_3$ | 36 | 1680 | 86, 87 (2), 102 (6) | 31 (2), 40 |
| 64 | $2 \times 4 : 2 : 2$ | 32 | 630 | 92, 96 (3), 97 (3) | 25 (2), 32 (2), 33, 41 |
| 65 | $D_8 \times 2 : 2$ | 32 | 315 | 98 (6), 93 | 34 (4), 43 (3) |
| 66 | $Q_8 : 2^2$ | 32 | 1890 | 98, 99, 95 (2), 91 (2), 94 | 35 (2), 43 |
| 67 | $Q_8 \times 2 : 2$ | 32 | 945 | 97 (4), 96, 94, 93 | 41, 42, 43 |
| 68 | $2 \times 8 \cdot 2$ | 32 | 945 | 99 (2), 93 | 42 (2), 43 |
| 69 | $D_8 : 2^2$ | 32 | 1890 | 96, 99, 89 (2), 90 (2), 98 | 43 |
| 70 | $15 : 2$ | 30 | 2016 | 100, 109, 128 (5) | 10 (2), 23, 47 |
| 71 | D_{30} | 30 | 2016 | 100, 110 (3), 129 (5) | 47 |
| 72 | $S_3 \times 5$ | 30 | 2016 | 100, 111 (3), 130 | 47 |
| 73 | 3^{1+2} | 27 | 1120 | 113, 112 (2), 114 | 48, 49, 50 |
| 74 | $2^2 : S_3$ | 24 | 5040 | 103, 118 (3), 125 (4) | 15, 24, 32 |
| 75 | $Q_8 : 3$ | 24 | 1260 | 115, 128 (4) | 19 (2), 51, 56, 57 |
| 76 | $A_4 \times 2$ | 24 | 5040 | 104, 119, 126 (4) | 30, 34, 38 |
| 77 | $A_4 \times 2$ | 24 | 2520 | 106, 120, 128 (4) | 33, 55 |
| 78 | $S_3 \times 2^2$ | 24 | 1680 | 108 (3), 105 (3), 107, 119 (3) | 35 (3), 38 |
| 79 | $7 : 3$ | 21 | 960 | 124, 137 (7) | 24 (3), 44, 58 (3) |
| 80 | $7 : 3$ | 21 | 1920 | 124, 138 (7) | 44 |
| 81 | 21 | 21 | 960 | 124, 138 | 44, 59 |
| 82 | D_{20} | 20 | 6048 | 111, 109, 110, 134 (5) | 8, 30, 47 |
| 83 | $S_3 \times 3$ | 18 | 3360 | 113, 125, 128 (3) | 17 (2), 23, 48, 60 |
| 84 | $S_3 \times 3$ | 18 | 6720 | 112, 127, 126 (3) | 29, 49 |
| 85 | $S_3 \times 3$ | 18 | 3360 | 113, 130, 126 (3) | 38 (2), 49, 60 |
| 86 | $3 : S_3$ | 18 | 280 | 114, 125 (12) | 48 (4), 61 (3), 63 (6) |
| 87 | $S_3 \times 3$ | 18 | 3360 | 114, 131, 126 (3) | 49, 50, 63 |
| 88 | $3 : S_3$ | 18 | 1120 | 113, 129 (9), 131 (3) | 50, 60 (3) |
| 89 | D_{16} | 16 | 3780 | 118, 116, 121 | 8 (2), 69 |
| 90 | D_{16} | 16 | 3780 | 121, 122, 118 | 16 (2), 69 |

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| Nr. | Structure | Order | Length | Maximal Subgroups | Minimal Overgroups |
| 91 | $Q_8 : 2$ | 16 | 3780 | 117, 116, 115 | 26 (2), 51, 66 |
| 92 | 2^4 | 16 | 42 | 120 (15) | 36 (6), 52, 54 (10), 55 (20), 64 (15) |
| 93 | 4^2 | 16 | 315 | 123 (3) | 53 (4), 65, 67 (3), 68 (3) |
| 94 | $Q_8 \times 2$ | 16 | 315 | 115 (4), 123 (3) | 56 (4), 66 (6), 67 (3) |
| 95 | $Q_8 : 2$ | 16 | 3780 | 117, 122, 115 | 57, 66 |
| 96 | $D_8 \times 2$ | 16 | 945 | 118 (4), 120 (2), 123 | 64 (2), 67, 69 (2) |
| 97 | $2 \times 4 : 2$ | 16 | 1890 | 120, 123 (2) | 64, 67 (2) |
| 98 | $D_8 \times 2$ | 16 | 1890 | 121 (2), 117 (2), 119 (2), 123 | 65, 66, 69 |
| 99 | 2×8 | 16 | 1890 | 116, 123, 122 | 66, 68, 69 |
| 100 | 15 | 15 | 2016 | 132, 139 | 18 (2), 70, 71, 72 |
| 101 | D_{14} | 14 | 2880 | 124, 140 (7) | 58, 59 |
| 102 | D_{12} | 12 | 10080 | 126, 131, 125, 134 (3) | 16, 30, 60, 63 |
| 103 | A_4 | 12 | 5040 | 135, 137 (4) | 45 (2), 54, 74 |
| 104 | A_4 | 12 | 1680 | 136, 137 (4) | 46, 53 (3), 54 (2), 62, 76 (3) |
| 105 | D_{12} | 12 | 5040 | 128, 130, 129, 134 (3) | 47 (2), 51, 60 (2), 78 |
| 106 | A_4 | 12 | 840 | 136, 139 (4) | 52, 62 (4), 77 (3) |
| 107 | 2×6 | 12 | 1680 | 128 (3), 136 | 55 (2), 56 (3), 62, 78 |
| 108 | D_{12} | 12 | 5040 | 128, 129 (2), 134 (3) | 57, 78 |
| 109 | D_{10} | 10 | 2016 | 132, 141 (5) | 25 (2), 45 (6), 46, 70, 82 (3) |
| 110 | D_{10} | 10 | 6048 | 132, 140 (5) | 71, 82 |
| 111 | 10 | 10 | 6048 | 132, 140 | 72, 82 |
| 112 | 3^2 | 9 | 2240 | 137, 138 (3) | 44 (3), 73, 84 (3) |
| 113 | 3^2 | 9 | 1120 | 139 (3), 137 | 62 (6), 73, 83 (3), 85 (3), 88 |
| 114 | 3^2 | 9 | 280 | 137 (4) | 73 (4), 86, 87 (12) |
| 115 | Q_8 | 8 | 1260 | 133 (3) | 37 (2), 75, 91 (3), 94, 95 (3) |
| 116 | 8 | 8 | 1890 | 133 | 39 (4), 89 (2), 91 (2), 99 |
| 117 | D_8 | 8 | 3780 | 134 (2), 133 | 40 (2), 91, 95, 98 |
| 118 | D_8 | 8 | 3780 | 135 (2), 133 | 74 (4), 89, 90, 96 |
| 119 | 2^3 | 8 | 1260 | 134 (6), 136 | 76 (4), 78 (4), 98 (3) |
| 120 | 2^3 | 8 | 630 | 135 (6), 136 | 77 (4), 92, 96 (3), 97 (3) |
| 121 | D_8 | 8 | 3780 | 134 (2), 133 | 89, 90, 98 |
| 122 | 8 | 8 | 1890 | 133 | 90 (2), 95 (2), 99 |
| 123 | 2×4 | 8 | 945 | 133 (2), 136 | 93, 94, 96, 97 (4), 98 (2), 99 (2) |

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| Nr. | Structure | Order | Length | Maximal Subgroups | Minimal Overgroups |
| 124 | 7 | 7 | 960 | 142 | 79, 80 (2), 81, 101 (3) |
| 125 | S_3 | 6 | 3360 | 137, 141 (3) | 45 (6), 46, 74 (6), 83, 86, 102 (3) |
| 126 | 6 | 6 | 10080 | 137, 140 | 58 (2), 76 (2), 84 (2), 85, 87, 102 |
| 127 | S_3 | 6 | 6720 | 138, 140 (3) | 59, 84 |
| 128 | 6 | 6 | 5040 | 139, 141 | 70 (2), 75, 77 (2), 83 (2), 105, 107, 108 |
| 129 | S_3 | 6 | 5040 | 139, 140 (3) | 71 (2), 88 (2), 105, 108 (2) |
| 130 | S_3 | 6 | 336 | 139, 140 (3) | 72 (6), 85 (10), 105 (15) |
| 131 | S_3 | 6 | 3360 | 137, 140 (3) | 87, 88, 102 (3) |
| 132 | 5 | 5 | 2016 | 142 | 36 (2), 100, 109, 110 (3), 111 (3) |
| 133 | 4 | 4 | 1890 | 141 | 61 (4), 115 (2), 116, 117 (2), 118 (2), 121 (2), 122, 123 |
| 134 | 2^2 | 4 | 7560 | 140 (2), 141 | 82 (4), 102 (4), 105 (2), 108 (2), 117, 119, 121 |
| 135 | 2^2 | 4 | 1260 | 141 (3) | 103 (4), 118 (6), 120 (3) |
| 136 | 2^2 | 4 | 105 | 141 (3) | 104 (16), 106 (8), 107 (16), 119 (12), 120 (6), 123 (9) |
| 137 | 3 | 3 | 1120 | 142 | 79 (6), 103 (18), 104 (6), 112 (2), 113, 114, 125 (3), 126 (9), 131 (3) |
| 138 | 3 | 3 | 960 | 142 | 80 (14), 81, 112 (7), 127 (7) |
| 139 | 3 | 3 | 336 | 142 | 100 (6), 106 (10), 113 (10), 128 (15), 129 (15), 130 |
| 140 | 2 | 2 | 1008 | 142 | 101 (20), 110 (30), 111 (6), 126 (10), 127 (20), 129 (15), 130, 131 (10), 134 (15) |
| 141 | 2 | 2 | 315 | 142 | 109 (32), 125 (32), 128 (16), 133 (6), 134 (24), 135 (12), 136 |

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| Nr. | Structure | Order | Length | Maximal Subgroups | Minimal Overgroups |
| 142 | 1 | 1 | 1 | | 124 (960), 132 (2016), 137 (1120), 138 (960), 139 (336), 140 (1008), 141 (315) |

TABLE 1. Subgroup lattice of $L_3(4) : S_3$

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