

THE SUBGROUP LATTICE OF $L_3(4) : 6$

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Nr.	Structure	Order	Length	Maximal Subgroups	Minimal Overgroups
1	$L_3(4) : 6$	120960	1	2, 3, 6 (105), 11 (280), 14 (336), 29 (960)	
2	$L_3(4) : 3$	60480	1	4, 5 (42), 19 (280), 44 (960)	1
3	$L_3(4) : 2$	40320	1	4, 8 (168), 12 (105), 16 (360), 26 (280), 30 (336)	1
4	$L_3(4)$	20160	1	7 (42), 15 (168), 24 (360), 37 (280)	2, 3
5	$2^4 : A_5 : 3$	2880	42	7, 9 (5), 10 (6), 17 (10), 23 (16)	2
6	$2 \times 4 : 2 : 2 : 2 : 3 : S_3$	1152	105	9, 12, 13 (3), 38 (16)	1
7	$2^4 : A_5$	960	42	20 (5), 25 (6), 32 (10), 45 (48), 46 (16)	4, 5
8	M_{10}	720	168	15, 39 (10), 84 (36), 92 (45)	3
9	$Q_8 \times 2 : 2 : 2 : 3 : 3$	576	105	21 (2), 22, 20, 27 (8)	5 (2), 6
10	$2^4 : 5 : 3 : 2$	480	252	18, 25, 33 (5), 73 (16)	5
11	$M_9 : 3 \times 2$	432	280	19, 26, 31 (4), 51 (9)	1
12	$2 \times 4 : 2 : 2 : 2 : S_3$	384	105	20, 28 (3), 34 (12)	3, 6
13	$2^{1+4} : A_4$	384	315	22, 28, 35 (4)	6
14	$S_5 \times 3$	360	336	23, 30, 38 (5), 47 (6), 62 (10)	1
15	A_6	360	168	45 (12), 63 (10), 75 (30)	4, 8
16	$L_2(7) : 2$	336	360	24, 58 (8), 93 (21), 105 (28)	3
17	$2^4 : 3 : S_3$	288	420	27, 32, 33 (3), 85 (16)	5
18	$2^4 : 5 : 3$	240	252	36, 52, 103 (16)	10
19	$M_9 : 3$	216	280	37, 48 (4), 76 (9)	2, 11
20	$2 \times 4 : 2 : 2 : 2 : 3$	192	105	41, 53 (12), 54 (8)	7 (2), 9, 12
21	$2^4 : 3 : 2 : 2$	192	210	33 (3), 41, 55 (4)	9

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Nr.	Structure	Order	Length	Maximal Subgroups	Minimal Overgroups
22	$Q_8 \times 2 : 2 : 2 : 3$	192	105	41, 56 (12), 55 (8)	9, 13 (3)
23	$A_5 \times 3$	180	336	46, 64 (5), 73 (6), 85 (10)	5 (2), 14
24	$L_2(7)$	168	360	75 (14), 80 (8)	4, 16
25	$2^4 : 5 : 2$	160	252	36, 67 (5), 112 (16)	7, 10
26	$3 : S_3 \cdot 2 \times 2 \cdot 2$	144	280	40 (3), 39 (3), 37, 94 (9)	3, 11
27	$A_4 \times 2 \times 2 : 3$	144	420	55 (2), 52, 54, 64 (8)	9 (2), 17
28	$2^{1+4} : 2^2$	128	315	41, 42 (3), 43 (3)	12, 13
29	$7 : 3 \times 3 : 2$	126	960	44, 59, 60, 61, 58, 86 (7)	1
30	S_5	120	336	46, 77 (5), 84 (6), 105 (10)	3, 14
31	$3^{1+2} : 2^2$	108	1120	49, 48, 50, 65, 62 (3)	11
32	$2^4 : S_3$	96	420	54, 67 (3), 75 (12)	7, 17
33	$2^4 : 3 : 2$	96	630	52, 67, 78 (4)	10 (2), 17 (2), 21
34	$4^2 : S_3$	96	1260	53, 68 (3), 77 (4)	12
35	$2^{1+4} : 3$	96	1260	57, 51, 56, 69, 79 (4)	13
36	$2^4 : 5$	80	252	95, 132 (16)	18, 25
37	M_9	72	280	63 (3), 116 (9)	4, 19, 26
38	$2 \times 6 : S_3$	72	1680	64, 77, 79 (3), 87 (4)	6, 14
39	M_9	72	840	66 (2), 63, 117 (9)	8 (2), 26
40	$3 : S_3 \cdot 2 \times 2$	72	840	66, 63, 65, 118 (9)	26
41	$2 \times 4 : 2 : 2 : 2$	64	105	67 (6), 70 (9)	20, 21 (2), 22, 28 (3)
42	$2^{1+4} : 2$	64	945	71 (2), 68 (2), 69, 70, 72	28
43	$Q_8 \times 2 \cdot 2 : 2$	64	945	70, 72 (2)	28
44	$7 : 3 \times 3$	63	960	81, 82, 83, 80, 113 (7)	2, 29
45	A_5	60	2016	106 (5), 112 (6), 126 (10)	7, 15
46	A_5	60	336	107 (5), 112 (6), 126 (10)	7 (2), 23, 30
47	$15 : 2 \cdot 2$	60	2016	73, 84, 108 (5)	14
48	$3^{1+2} : 2$	54	1120	74, 88, 85 (3)	19, 31
49	$3^{1+2} : 2$	54	1120	74, 89, 87 (3)	31
50	$3^{1+2} \times 2$	54	1120	74, 90, 86 (2), 91	31
51	$Q_8 : 6$	48	1260	76, 94, 109 (4)	11 (2), 35
52	$2^4 : 3$	48	42	95, 110 (20)	18 (6), 27 (10), 33 (15)
53	$4^2 : 3$	48	1260	96, 107 (4)	20, 34
54	$2^4 : 3$	48	420	95, 106 (12), 107 (8)	20 (2), 27, 32
55	$A_4 \times 2 \times 2$	48	840	78 (3), 95, 111 (4)	21, 22, 27
56	$Q_8 : 6$	48	1260	76, 97, 111 (4)	22, 35

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Nr.	Structure	Order	Length	Maximal Subgroups	Minimal Overgroups
57	$Q_8 \cdot 6$	48	1260	76, 98, 108 (4)	35
58	$7 : 3 : 2$	42	960	80, 104, 127 (7)	16 (3), 29
59	$7 : 3 : 2$	42	960	81, 104, 128 (7)	29
60	$7 : 3 : 2$	42	960	82, 104, 128 (7)	29
61	$21 : 2$	42	960	83, 104, 128 (7)	29
62	$S_3 \times 6$	36	3360	90, 87, 85, 105, 109 (3)	14, 31
63	$3 : S_3 \cdot 2$	36	840	88, 133 (9)	15 (2), 37, 39, 40
64	$A_4 \times 3$	36	1680	107, 111, 110 (2), 114 (4)	23, 27 (2), 38
65	$6 : S_3$	36	280	89, 91, 88, 105 (12)	31 (4), 40 (3)
66	$3 : S_3 \cdot 2$	36	840	88, 134 (9)	39 (2), 40
67	$D_8 \times 2 : 2$	32	630	95, 99 (3), 100 (3)	25 (2), 32 (2), 33, 41
68	$D_8 : 4$	32	1890	101, 102, 96	34 (2), 42
69	2^{1+4}	32	315	98 (4), 94 (4), 101 (6), 97	35 (4), 42 (3)
70	$Q_8 \times 2 : 2$	32	945	100 (4), 99, 97, 96	41, 42, 43
71	$Q_8 : 2^2$	32	1890	93 (2), 92 (2), 101, 99, 102	42
72	$Q_8 \times 2 \cdot 2$	32	945	102 (2), 97	42, 43 (2)
73	$15 : 2$	30	2016	103, 112, 129 (5)	10 (2), 23, 47
74	3^{1+2}	27	1120	114, 113 (2), 115	48, 49, 50
75	$2^2 : S_3$	24	5040	106, 119 (3), 126 (4)	15, 24, 32
76	$Q_8 : 3$	24	1260	116, 129 (4)	19 (2), 51, 56, 57
77	$2^2 : S_3$	24	1680	107, 120 (3), 130 (4)	30, 34 (3), 38
78	$A_4 \times 2$	24	2520	110, 121, 129 (4)	33, 55
79	$D_8 \times 3$	24	5040	111, 108, 109, 120	35, 38
80	$7 : 3$	21	960	125, 138 (7)	24 (3), 44, 58
81	$7 : 3$	21	960	125, 139 (7)	44, 59
82	$7 : 3$	21	960	125, 139 (7)	44, 60
83	21	21	960	125, 139	44, 61
84	$D_{10} \cdot 2$	20	2016	112, 134 (5)	8 (3), 30, 47
85	$S_3 \times 3$	18	3360	114, 126, 129 (3)	17 (2), 23, 48, 62
86	3×6	18	2240	113, 127, 128 (3)	29 (3), 50
87	$S_3 \times 3$	18	3360	114, 130, 131 (3)	38 (2), 49, 62
88	$3 : S_3$	18	280	115, 126 (12)	48 (4), 63 (3), 65, 66 (3)
89	$3 : S_3$	18	280	115, 130 (12)	49 (4), 65
90	3×6	18	1120	114, 131 (3), 127	50, 62 (3)
91	3×6	18	280	115, 127 (4)	50 (4), 65
92	$Q_8 : 2$	16	3780	119, 122, 117	8 (2), 71
93	D_{16}	16	3780	123, 122, 119	16 (2), 71
94	$Q_8 \times 2$	16	1260	118 (3), 117 (3), 116	26 (2), 51, 69

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Nr.	Structure	Order	Length	Maximal Subgroups	Minimal Overgroups
95	2^4	16	42	121 (15)	36 (6), 52, 54 (10), 55 (20), 67 (15)
96	4^2	16	315	124 (3)	53 (4), 68 (6), 70 (3)
97	$Q_8 \times 2$	16	315	116 (4), 124 (3)	56 (4), 69, 70 (3), 72 (3)
98	$D_8 : 2$	16	1260	123 (3), 118 (3), 116	57, 69
99	$D_8 \times 2$	16	945	119 (4), 121 (2), 124	67 (2), 70, 71 (2)
100	$2 \times 4 : 2$	16	1890	121, 124 (2)	67, 70 (2)
101	$D_8 : 2$	16	1890	120 (2), 118 (2), 123, 117, 124	68, 69, 71
102	$8 : 2$	16	1890	124, 122 (2)	68, 71, 72
103	15	15	2016	132, 140	18 (2), 73
104	D_{14}	14	960	125, 141 (7)	58, 59, 60, 61
105	D_{12}	12	3360	127, 130, 126, 135 (3)	16 (3), 30, 62, 65
106	A_4	12	5040	136, 138 (4)	45 (2), 54, 75
107	A_4	12	1680	137, 138 (4)	46, 53 (3), 54 (2), 64, 77
108	12	12	5040	129, 134	47 (2), 57, 79
109	2×6	12	5040	131 (2), 129, 135	51, 62 (2), 79
110	A_4	12	840	137, 140 (4)	52, 64 (4), 78 (3)
111	2×6	12	1680	129 (3), 137	55 (2), 56 (3), 64, 79 (3)
112	D_{10}	10	2016	132, 142 (5)	25 (2), 45 (6), 46, 73, 84
113	3^2	9	2240	138, 139 (3)	44 (3), 74, 86
114	3^2	9	1120	140 (3), 138	64 (6), 74, 85 (3), 87 (3), 90
115	3^2	9	280	138 (4)	74 (4), 88, 89, 91
116	Q_8	8	1260	133 (3)	37 (2), 76, 94, 97, 98
117	Q_8	8	1890	133, 134 (2)	39 (4), 92 (2), 94 (2), 101
118	2×4	8	3780	135, 134, 133	40 (2), 94, 98, 101
119	D_8	8	3780	136 (2), 133	75 (4), 92, 93, 99
120	D_8	8	1260	135, 134, 137	77 (4), 79 (4), 101 (3)
121	2^3	8	630	136 (6), 137	78 (4), 95, 99 (3), 100 (3)
122	8	8	3780	133	92, 93, 102
123	D_8	8	1890	135 (2), 133	93 (2), 98 (2), 101
124	2×4	8	945	133 (2), 137	96, 97, 99, 100 (4), 101 (2), 102 (2)
125	7	7	960	143	80, 81, 82, 83, 104
126	S_3	6	3360	138, 142 (3)	45 (6), 46, 75 (6), 85, 88, 105

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127	6	6	1120	138, 141	58 (6), 86 (2), 90, 91, 105 (3)
128	6	6	6720	139, 141	59, 60, 61, 86
129	6	6	5040	140, 142	73 (2), 76, 78 (2), 85 (2), 108, 109, 111
130	S_3	6	3360	138, 141 (3)	77 (2), 87, 89, 105
131	6	6	3360	140, 141	87 (3), 90, 109 (3)
132	5	5	2016	143	36 (2), 103, 112
133	4	4	1890	142	63 (4), 116 (2), 117, 118 (2), 119 (2), 122 (2), 123, 124
134	4	4	1260	142	66 (6), 84 (8), 108 (4), 117 (3), 118 (3), 120
135	2^2	4	1260	142, 141 (2)	105 (8), 109 (4), 118 (3), 120, 123 (3)
136	2^2	4	1260	142 (3)	106 (4), 119 (6), 121 (3)
137	2^2	4	105	142 (3)	107 (16), 110 (8), 111 (16), 120 (12), 121 (6), 124 (9)
138	3	3	1120	143	80 (6), 106 (18), 107 (6), 113 (2), 114, 115, 126 (3), 127, 130 (3)
139	3	3	960	143	81 (7), 82 (7), 83, 113 (7), 128 (7)
140	3	3	336	143	103 (6), 110 (10), 114 (10), 129 (15), 131 (10)
141	2	2	280	143	104 (24), 127 (4), 128 (24), 130 (36), 131 (12), 135 (9)
142	2	2	315	143	112 (32), 126 (32), 129 (16), 133 (6), 134 (4), 135 (4), 136 (12), 137
143	1	1	1		125 (960), 132 (2016), 138 (1120), 139 (960), 140 (336), 141 (280), 142 (315)

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TABLE 1. Subgroup lattice of $L_3(4) : 6$

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