

THE SUBGROUP LATTICE OF $L_3(4) : 2$

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Nr.	Structure	Order	Length	Maximal Subgroups	Minimal Overgroups
1	$L_3(4) : 2$	40320	1	2, 4 (56), 5 (56), 6 (56), 7 (105), 11 (120), 12 (120), 13 (120), 19 (280), 21 (336)	
2	$L_3(4)$	20160	1	3 (42), 8 (56), 9 (56), 10 (56), 15 (120), 16 (120), 17 (120), 27 (280)	1
3	$2^4 : A_5$	960	42	14 (5), 18 (6), 22 (10), 41 (16), 42 (16), 43 (16), 44 (16)	2
4	M_{10}	720	56	10, 28 (10), 76 (36), 80 (45)	1
5	M_{10}	720	56	9, 29 (10), 76 (36), 81 (45)	1
6	M_{10}	720	56	8, 30 (10), 76 (36), 82 (45)	1
7	$2 \times 4 : 2 : 2 : 2 : S_3$	384	105	14, 20 (3), 23 (4), 24 (4), 25 (4)	1
8	A_6	360	56	44 (12), 50 (10), 71 (30)	2, 6
9	A_6	360	56	42 (12), 51 (10), 72 (30)	2, 5
10	A_6	360	56	41 (12), 52 (10), 73 (30)	2, 4
11	$L_2(7) : 2$	336	120	15, 49 (8), 83 (21), 106 (28)	1
12	$L_2(7) : 2$	336	120	16, 49 (8), 84 (21), 106 (28)	1
13	$L_2(7) : 2$	336	120	17, 49 (8), 85 (21), 106 (28)	1
14	$2 \times 4 : 2 : 2 : 2 : 3$	192	105	34, 45 (4), 46 (8), 47 (4), 48 (4)	3 (2), 7

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Nr.	Structure	Order	Length	Maximal Subgroups	Minimal Overgroups
15	$L_2(7)$	168	120	72 (14), 75 (8)	2, 11
16	$L_2(7)$	168	120	71 (14), 75 (8)	2, 12
17	$L_2(7)$	168	120	73 (14), 75 (8)	2, 13
18	$2^4 : 5 : 2$	160	252	26, 57 (5), 111 (16)	3
19	$3 : S_3 \cdot 2 \times 2 \cdot 2$	144	280	31, 32, 30, 33, 28, 27, 29, 86 (9)	1
20	$2^{1+4} : 2^2$	128	315	34, 35, 36, 37, 38, 39, 40	7
21	S_5	120	336	43, 74 (5), 76 (6), 106 (10)	1
22	$2^4 : S_3$	96	420	46, 57 (3), 72 (4), 71 (4), 73 (4)	3
23	$4^2 : S_3$	96	420	48, 58 (3), 74 (4)	7
24	$4^2 : S_3$	96	420	47, 59 (3), 74 (4)	7
25	$4^2 : S_3$	96	420	45, 60 (3), 74 (4)	7
26	$2^4 : 5$	80	252	87, 138 (16)	18
27	M_9	72	280	51, 50, 52, 113 (9)	2, 19
28	M_9	72	280	53, 54, 52, 114 (9)	4 (2), 19
29	M_9	72	280	53, 55, 51, 115 (9)	5 (2), 19
30	M_9	72	280	55, 54, 50, 116 (9)	6 (2), 19
31	$3 : S_3 \cdot 2 \times 2$	72	280	55, 52, 56, 117 (9)	19
32	$3 : S_3 \cdot 2 \times 2$	72	280	54, 51, 56, 118 (9)	19
33	$3 : S_3 \cdot 2 \times 2$	72	280	53, 50, 56, 119 (9)	19
34	$2 \times 4 : 2 : 2 : 2$	64	105	57 (6), 61 (3), 62 (3), 63 (3)	14, 20 (3)
35	$2^{1+4} : 2$	64	315	64 (2), 58 (2), 65, 62, 66	20
36	$Q_8 \times 2 \cdot 2 : 2$	64	315	61, 67, 66	20
37	$2^{1+4} : 2$	64	315	65, 67, 60 (2), 68 (2), 63	20
38	$2^{1+4} : 2$	64	315	69 (2), 59 (2), 65, 61, 70	20
39	$Q_8 \times 2 \cdot 2 : 2$	64	315	62, 67, 70	20
40	$Q_8 \times 2 \cdot 2 : 2$	64	315	63, 66, 70	20
41	A_5	60	672	107 (5), 111 (6), 135 (10)	3, 10
42	A_5	60	672	108 (5), 111 (6), 135 (10)	3, 9
43	A_5	60	336	109 (5), 111 (6), 135 (10)	3 (2), 21
44	A_5	60	672	110 (5), 111 (6), 135 (10)	3, 8

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Nr.	Structure	Order	Length	Maximal Subgroups	Minimal Overgroups
45	$4^2 : 3$	48	420	88, 109 (4)	14, 25
46	$2^4 : 3$	48	420	87, 109 (8), 108 (4), 110 (4), 107 (4)	14 (2), 22
47	$4^2 : 3$	48	420	89, 109 (4)	14, 24
48	$4^2 : 3$	48	420	90, 109 (4)	14, 23
49	$7 : 3 : 2$	42	960	75, 105, 136 (7)	11, 12, 13
50	$3 : S_3 \cdot 2$	36	280	77, 139 (9)	8 (2), 27, 30, 33
51	$3 : S_3 \cdot 2$	36	280	77, 140 (9)	9 (2), 27, 29, 32
52	$3 : S_3 \cdot 2$	36	280	77, 141 (9)	10 (2), 27, 28, 31
53	$3 : S_3 \cdot 2$	36	280	77, 142 (9)	28, 29, 33
54	$3 : S_3 \cdot 2$	36	280	77, 142 (9)	28, 30, 32
55	$3 : S_3 \cdot 2$	36	280	77, 142 (9)	29, 30, 31
56	$6 : S_3$	36	280	78, 79, 77, 106 (12)	31, 32, 33
57	$D_8 \times 2 : 2$	32	630	87, 91, 92, 93, 94, 95, 96	18 (2), 22 (2), 34
58	$D_8 : 4$	32	630	97, 98, 90	23 (2), 35
59	$D_8 : 4$	32	630	99, 100, 89	24 (2), 38
60	$D_8 : 4$	32	630	101, 102, 88	25 (2), 37
61	$4^2 : 2$	32	315	92 (2), 96, 89, 91 (2), 103	34, 36, 38
62	$2 \times 4 : 2 : 2$	32	315	93 (2), 91 (2), 94, 90, 103	34, 35, 39
63	$Q_8 \times 2 : 2$	32	315	92 (2), 93 (2), 95, 103, 88	34, 37, 40
64	$Q_8 : 2^2$	32	630	84 (2), 82 (2), 97, 94, 98	35
65	2^{1+4}	32	315	104 (4), 86 (4), 101 (2), 99 (2), 97 (2), 103	35, 37, 38
66	$Q_8 \times 2 \cdot 2$	32	315	98 (2), 103	35, 36, 40
67	$Q_8 \times 2 \cdot 2$	32	315	102 (2), 103	36, 37, 39
68	$D_8 \cdot 2^2$	32	630	85 (2), 80 (2), 101, 102, 95	37
69	$D_8 \cdot 2^2$	32	630	83 (2), 81 (2), 99, 100, 96	38
70	$8 : 2 \cdot 2$	32	315	100 (2), 103	38, 39, 40
71	$2^2 : S_3$	24	1680	110, 120 (3), 135 (4)	8, 16, 22
72	$2^2 : S_3$	24	1680	108, 121 (3), 135 (4)	9, 15, 22
73	$2^2 : S_3$	24	1680	107, 122 (3), 135 (4)	10, 17, 22
74	$2^2 : S_3$	24	1680	109, 123 (3), 137 (4)	21, 23, 24, 25
75	$7 : 3$	21	960	134, 148 (7)	15, 16, 17, 49
76	$D_{10} \cdot 2$	20	2016	111, 142 (5)	4, 5, 6, 21
77	$3 : S_3$	18	280	112, 135 (12)	50, 51, 52, 53, 54, 55, 56

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Nr.	Structure	Order	Length	Maximal Subgroups	Minimal Overgroups
78	$3 : S_3$	18	280	112, 137 (12)	56
79	3×6	18	280	112, 136 (4)	56
80	$Q_8 : 2$	16	1260	122, 124, 114	4 (2), 68
81	$Q_8 : 2$	16	1260	121, 125, 115	5 (2), 69
82	$Q_8 : 2$	16	1260	120, 126, 116	6 (2), 64
83	D_{16}	16	1260	127, 125, 121	11 (2), 69
84	D_{16}	16	1260	128, 126, 120	12 (2), 64
85	D_{16}	16	1260	129, 124, 122	13 (2), 68
86	$Q_8 \times 2$	16	1260	119, 115, 118, 116, 117, 114, 113	19 (2), 65
87	2^4	16	42	130 (15)	26 (6), 46 (10), 57 (15)
88	4^2	16	105	131 (3)	45 (4), 60 (6), 63 (3)
89	4^2	16	105	132 (3)	47 (4), 59 (6), 61 (3)
90	4^2	16	105	133 (3)	48 (4), 58 (6), 62 (3)
91	$2 \times 4 : 2$	16	630	130, 132, 133	57, 61, 62
92	$2 \times 4 : 2$	16	630	130, 131, 132	57, 61, 63
93	$2 \times 4 : 2$	16	630	130, 131, 133	57, 62, 63
94	$D_8 \times 2$	16	315	120 (4), 130 (2), 133	57 (2), 62, 64 (2)
95	$D_8 \times 2$	16	315	122 (4), 130 (2), 131	57 (2), 63, 68 (2)
96	$D_8 \times 2$	16	315	121 (4), 130 (2), 132	57 (2), 61, 69 (2)
97	$D_8 : 2$	16	630	128, 116, 119 (2), 123 (2), 133	58, 64, 65
98	$8 : 2$	16	630	133, 126 (2)	58, 64, 66
99	$D_8 : 2$	16	630	127, 115, 118 (2), 123 (2), 132	59, 65, 69
100	$8 : 2$	16	630	132, 125 (2)	59, 69, 70
101	$D_8 : 2$	16	630	123 (2), 117 (2), 129, 114, 131	60, 65, 68
102	$8 : 2$	16	630	131, 124 (2)	60, 67, 68
103	$Q_8 \times 2$	16	315	113 (4), 131, 132, 133	61, 62, 63, 65, 66, 67, 70
104	$D_8 : 2$	16	1260	129, 127, 118, 117, 128, 119, 113	65
105	D_{14}	14	960	134, 149 (7)	49
106	D_{12}	12	3360	136, 137, 135, 143 (3)	11, 12, 13, 21, 56
107	A_4	12	1680	144, 148 (4)	41 (2), 46, 73
108	A_4	12	1680	145, 148 (4)	42 (2), 46, 72
109	A_4	12	1680	146, 148 (4)	43, 45, 46 (2), 47, 48, 74
110	A_4	12	1680	147, 148 (4)	44 (2), 46, 71
111	D_{10}	10	2016	138, 150 (5)	18 (2), 41 (2), 42 (2), 43, 44 (2), 76

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Nr.	Structure	Order	Length	Maximal Subgroups	Minimal Overgroups
112	3^2	9	280	148 (4)	77, 78, 79
113	Q_8	8	1260	141, 140, 139	27 (2), 86, 103, 104
114	Q_8	8	630	141, 142 (2)	28 (4), 80 (2), 86 (2), 101
115	Q_8	8	630	140, 142 (2)	29 (4), 81 (2), 86 (2), 99
116	Q_8	8	630	139, 142 (2)	30 (4), 82 (2), 86 (2), 97
117	2×4	8	1260	143, 142, 141	31 (2), 86, 101, 104
118	2×4	8	1260	143, 142, 140	32 (2), 86, 99, 104
119	2×4	8	1260	143, 142, 139	33 (2), 86, 97, 104
120	D_8	8	1260	147 (2), 139	71 (4), 82, 84, 94
121	D_8	8	1260	145 (2), 140	72 (4), 81, 83, 96
122	D_8	8	1260	144 (2), 141	73 (4), 80, 85, 95
123	D_8	8	1260	143, 142, 146	74 (4), 97, 99, 101
124	8	8	1260	141	80, 85, 102
125	8	8	1260	140	81, 83, 100
126	8	8	1260	139	82, 84, 98
127	D_8	8	630	143 (2), 140	83 (2), 99, 104 (2)
128	D_8	8	630	143 (2), 139	84 (2), 97, 104 (2)
129	D_8	8	630	143 (2), 141	85 (2), 101, 104 (2)
130	2^3	8	630	146, 144 (2), 145 (2), 147 (2)	87, 91, 92, 93, 94, 95, 96
131	2×4	8	315	141 (2), 146	88, 92 (2), 93 (2), 95, 101 (2), 102 (2), 103
132	2×4	8	315	140 (2), 146	89, 91 (2), 92 (2), 96, 99 (2), 100 (2), 103
133	2×4	8	315	139 (2), 146	90, 91 (2), 93 (2), 94, 97 (2), 98 (2), 103
134	7	7	960	151	75, 105
135	S_3	6	3360	148, 150 (3)	41 (2), 42 (2), 43, 44 (2), 71 (2), 72 (2), 73 (2), 77, 106
136	6	6	1120	148, 149	49 (6), 79, 106 (3)
137	S_3	6	3360	148, 149 (3)	74 (2), 78, 106
138	5	5	2016	151	26 (2), 111
139	4	4	630	150	50 (4), 113 (2), 116, 119 (2), 120 (2), 126 (2), 128, 133
140	4	4	630	150	51 (4), 113 (2), 115, 118 (2), 121 (2), 125 (2), 127, 132

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Nr.	Structure	Order	Length	Maximal Subgroups	Minimal Overgroups
141	4	4	630	150	52 (4), 113 (2), 114, 117 (2), 122 (2), 124 (2), 129, 131
142	4	4	1260	150	53 (2), 54 (2), 55 (2), 76 (8), 114, 115, 116, 117, 118, 119, 123
143	2^2	4	1260	150, 149 (2)	106 (8), 117, 118, 119, 123, 127, 128, 129
144	2^2	4	420	150 (3)	107 (4), 122 (6), 130 (3)
145	2^2	4	420	150 (3)	108 (4), 121 (6), 130 (3)
146	2^2	4	105	150 (3)	109 (16), 123 (12), 130 (6), 131 (3), 132 (3), 133 (3)
147	2^2	4	420	150 (3)	110 (4), 120 (6), 130 (3)
148	3	3	1120	151	75 (6), 107 (6), 108 (6), 109 (6), 110 (6), 112, 135 (3), 136, 137 (3)
149	2	2	280	151	105 (24), 136 (4), 137 (36), 143 (9)
150	2	2	315	151	111 (32), 135 (32), 139 (2), 140 (2), 141 (2), 142 (4), 143 (4), 144 (4), 145 (4), 146, 147 (4)
151	1	1	1		134 (960), 138 (2016), 148 (1120), 149 (280), 150 (315)

TABLE 1. Subgroup lattice of $L_3(4) : 2$

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