

THE SUBGROUP LATTICE OF $L_3(4)$

THOMAS CONNOR AND DIMITRI LEEMANS

Nr.	Structure	Order	Length	Maximal Subgroups	Minimal Overgroups
1	$L_3(4)$	20160	1	2 (21), 3 (21), 4 (56), 5 (56), 6 (56), 8 (120), 9 (120), 10 (120), 17 (280)	
2	$2^4 : A_5$	960	21	7 (5), 11 (6), 13 (10), 19 (16), 20 (16), 21 (16), 22 (16)	1
3	$2^4 : A_5$	960	21	7 (5), 12 (6), 14 (10), 20 (16), 23 (16), 24 (16), 25 (16)	1
4	A_6	360	56	21 (6), 24 (6), 31 (10), 39 (15), 40 (15)	1
5	A_6	360	56	25 (6), 19 (6), 32 (10), 41 (15), 42 (15)	1
6	A_6	360	56	23 (6), 22 (6), 33 (10), 43 (15), 44 (15)	1
7	$2^4 : 2 : 2 : 3$	192	105	18, 26 (4), 27 (4), 28 (4), 29 (4), 30 (4)	2, 3
8	$L_2(7)$	168	120	42 (7), 41 (7), 45 (8)	1
9	$L_2(7)$	168	120	39 (7), 40 (7), 45 (8)	1
10	$L_2(7)$	168	120	43 (7), 44 (7), 45 (8)	1
11	$2^4 : 5 : 2$	160	126	15, 34 (5), 69 (16)	2
12	$2^4 : 5 : 2$	160	126	16, 35 (5), 69 (16)	3
13	$2^4 : S_3$	96	210	28, 34 (3), 39 (4), 42 (4), 44 (4)	2
14	$2^4 : S_3$	96	210	27, 35 (3), 43 (4), 41 (4), 40 (4)	3
15	$2^4 : 5$	80	126	47, 82 (16)	11
16	$2^4 : 5$	80	126	48, 82 (16)	12
17	M_9	72	280	32, 31, 33, 71 (9)	1
18	$2^4 : 2 : 2$	64	105	34 (3), 35 (3), 36 (3), 37 (3), 38 (3)	7
19	A_5	60	336	62 (5), 69 (6), 81 (10)	2, 5
20	A_5	60	336	63 (5), 69 (6), 81 (10)	2, 3

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Nr.	Structure	Order	Length	Maximal Subgroups	Minimal Overgroups
21	A_5	60	336	64 (5), 69 (6), 81 (10)	2, 4
22	A_5	60	336	65 (5), 69 (6), 81 (10)	2, 6
23	A_5	60	336	66 (5), 69 (6), 81 (10)	3, 6
24	A_5	60	336	67 (5), 69 (6), 81 (10)	3, 4
25	A_5	60	336	68 (5), 69 (6), 81 (10)	3, 5
26	$4^2 : 3$	48	420	49, 63 (4)	7
27	$2^4 : 3$	48	210	48, 63 (8), 62 (4), 65 (4), 64 (4)	7 (2), 14
28	$2^4 : 3$	48	210	47, 66 (4), 67 (4), 63 (8), 68 (4)	7 (2), 13
29	$4^2 : 3$	48	420	50, 63 (4)	7
30	$4^2 : 3$	48	420	51, 63 (4)	7
31	$3 : S_3 \cdot 2$	36	280	46, 83 (9)	4 (2), 17
32	$3 : S_3 \cdot 2$	36	280	46, 84 (9)	5 (2), 17
33	$3 : S_3 \cdot 2$	36	280	46, 85 (9)	6 (2), 17
34	$D_8 \times 2 : 2$	32	315	47, 52, 53, 54, 55, 56, 57	11 (2), 13 (2), 18
35	$2^3 \cdot 2 : 2$	32	315	56, 58, 55, 59, 57, 60, 48	12 (2), 14 (2), 18
36	$4^2 : 2$	32	315	54, 61, 52, 50, 55, 60, 59	18
37	$2 \times 4 : 2 : 2$	32	315	54, 58, 53, 60, 56, 51, 61	18
38	$2 \times 4 : 2 : 2$	32	315	57, 61, 52, 58, 53, 49, 59	18
39	$2^2 : S_3$	24	840	67, 72 (3), 81 (4)	4, 9, 13
40	$2^2 : S_3$	24	840	64, 72 (3), 81 (4)	4, 9, 14
41	$2^2 : S_3$	24	840	62, 73 (3), 81 (4)	5, 8, 14
42	$2^2 : S_3$	24	840	68, 73 (3), 81 (4)	5, 8, 13
43	$2^2 : S_3$	24	840	65, 74 (3), 81 (4)	6, 10, 14
44	$2^2 : S_3$	24	840	66, 74 (3), 81 (4)	6, 10, 13
45	$7 : 3$	21	960	80, 93 (7)	8, 9, 10
46	$3 : S_3$	18	280	70, 81 (12)	31, 32, 33
47	2^4	16	21	75 (15)	15 (6), 28 (10), 34 (15)
48	2^4	16	21	76 (15)	16 (6), 27 (10), 35 (15)
49	4^2	16	105	77 (3)	26 (4), 38 (3)
50	4^2	16	105	78 (3)	29 (4), 36 (3)
51	4^2	16	105	79 (3)	30 (4), 37 (3)
52	$2 \times 4 : 2$	16	315	75, 77, 78	34, 36, 38
53	$2 \times 4 : 2$	16	315	75, 79, 77	34, 37, 38
54	$2 \times 4 : 2$	16	315	75, 79, 78	34, 36, 37

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Nr.	Structure	Order	Length	Maximal Subgroups	Minimal Overgroups
55	$D_8 \times 2$	16	315	73 (4), 75, 78, 76	34, 35, 36
56	$D_8 \times 2$	16	315	74 (4), 75, 79, 76	34, 35, 37
57	$D_8 \times 2$	16	315	72 (4), 75, 77, 76	34, 35, 38
58	$2 \times 4 : 2$	16	315	76, 79, 77	35, 37, 38
59	$2 \times 4 : 2$	16	315	76, 78, 77	35, 36, 38
60	$2 \times 4 : 2$	16	315	76, 78, 79	35, 36, 37
61	$Q_8 \times 2$	16	315	71 (4), 78, 79, 77	36, 37, 38
62	A_4	12	840	86, 93 (4)	19 (2), 27, 41
63	A_4	12	1680	87, 93 (4)	20, 26, 27, 28, 29, 30
64	A_4	12	840	88, 93 (4)	21 (2), 27, 40
65	A_4	12	840	89, 93 (4)	22 (2), 27, 43
66	A_4	12	840	90, 93 (4)	23 (2), 28, 44
67	A_4	12	840	91, 93 (4)	24 (2), 28, 39
68	A_4	12	840	92, 93 (4)	25 (2), 28, 42
69	D_{10}	10	2016	82, 94 (5)	11, 12, 19, 20, 21, 22, 23, 24, 25
70	3^2	9	280	93 (4)	46
71	Q_8	8	1260	85, 84, 83	17 (2), 61
72	D_8	8	1260	91, 83, 88	39 (2), 40 (2), 57
73	D_8	8	1260	92, 84, 86	41 (2), 42 (2), 55
74	D_8	8	1260	90, 85, 89	43 (2), 44 (2), 56
75	2^3	8	315	87, 91 (2), 90 (2), 92 (2)	47, 52, 53, 54, 55, 56, 57
76	2^3	8	315	86 (2), 89 (2), 88 (2), 87	48, 55, 56, 57, 58, 59, 60
77	2×4	8	315	83 (2), 87	49, 52, 53, 57, 58, 59, 61
78	2×4	8	315	84 (2), 87	50, 52, 54, 55, 59, 60, 61
79	2×4	8	315	85 (2), 87	51, 53, 54, 56, 58, 60, 61
80	7	7	960	95	45
81	S_3	6	3360	93, 94 (3)	19, 20, 21, 22, 23, 24, 25, 39, 40, 41, 42, 43, 44, 46
82	5	5	2016	95	15, 16, 69
83	4	4	630	94	31 (4), 71 (2), 72 (2), 77
84	4	4	630	94	32 (4), 71 (2), 73 (2), 78
85	4	4	630	94	33 (4), 71 (2), 74 (2), 79

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Nr.	Structure	Order	Length	Maximal Subgroups	Minimal Overgroups
86	2^2	4	210	94 (3)	62 (4), 73 (6), 76 (3)
87	2^2	4	105	94 (3)	63 (16), 75 (3), 76 (3), 77 (3), 78 (3), 79 (3)
88	2^2	4	210	94 (3)	64 (4), 72 (6), 76 (3)
89	2^2	4	210	94 (3)	65 (4), 74 (6), 76 (3)
90	2^2	4	210	94 (3)	66 (4), 74 (6), 75 (3)
91	2^2	4	210	94 (3)	67 (4), 72 (6), 75 (3)
92	2^2	4	210	94 (3)	68 (4), 73 (6), 75 (3)
93	3	3	1120	95	45 (6), 62 (3), 63 (6), 64 (3), 65 (3), 66 (3), 67 (3), 68 (3), 70, 81 (3)
94	2	2	315	95	69 (32), 81 (32), 83 (2), 84 (2), 85 (2), 86 (2), 87, 88 (2), 89 (2), 90 (2), 91 (2), 92 (2)
95	1	1	1		80 (960), 82 (2016), 93 (1120), 94 (315)

TABLE 1. Subgroup lattice of $L_3(4)$

UNIVERSITÉ LIBRE DE BRUXELLES, DÉPARTEMENT DE MATHÉMATIQUES - C.P.216, BOULEVARD DU TRIOMPHE,
 B-1050 BRUXELLES, BOURSIER FR1A
E-mail address: tconnor@ulb.ac.be

UNIVERSITY OF AUCKLAND, DEPARTMENT OF MATHEMATICS, PRIVATE BAG 92019, AUCKLAND, NEW ZEALAND
E-mail address: d.leemans@auckland.ac.nz