

THE SUBGROUP LATTICE OF $L_3(3) : 2$

THOMAS CONNOR AND DIMITRI LEEMANS

| Nr. | Structure | Order | Length | Maximal Subgroups | Minimal Overgroups |
|-----|---------------------------|-------|--------|---|--------------------|
| 1 | $L_3(3) : 2$ | 11232 | 1 | 2, 4 (52), 10 (117), 11 (144), 18 (234) | |
| 2 | $L_3(3)$ | 5616 | 1 | 3 (26), 22 (144), 29 (234) | 1 |
| 3 | $M_9 : S_3$ | 432 | 26 | 5, 6 (3), 7 (4), 19 (9) | 2 |
| 4 | $3^{1+2} : D_8$ | 216 | 52 | 8, 9, 7, 30 (9) | 1 |
| 5 | $M_9 : 3$ | 216 | 26 | 12, 15 (4), 31 (9) | 3 |
| 6 | $M_9 : 2$ | 144 | 78 | 13, 12, 14, 39 (9) | 3 |
| 7 | $3^{1+2} : 2^2$ | 108 | 52 | 15 (2), 16, 23 (6) | 3 (2), 4 |
| 8 | $3^{1+2} : 2^2$ | 108 | 52 | 17 (2), 16, 24 (6) | 4 |
| 9 | $3^{1+2} : 4$ | 108 | 52 | 16, 47 (9) | 4 |
| 10 | $D_8 : 2 : S_3$ | 96 | 117 | 20, 19, 21, 26 (3), 30 (4) | 1 |
| 11 | $13 : 3 : 2$ | 78 | 144 | 22, 28, 63 (13) | 1 |
| 12 | M_9 | 72 | 26 | 25 (3), 54 (9) | 5, 6 (3) |
| 13 | $3 : S_3 \cdot 2 \cdot 2$ | 72 | 78 | 25, 55 (9) | 6 |
| 14 | $3 : S_3 \cdot 2 : 2$ | 72 | 78 | 23 (2), 25, 56 (9) | 6 |
| 15 | $3^{1+2} : 2$ | 54 | 104 | 27, 34, 35 (3) | 5, 7 |
| 16 | $3^{1+2} : 2$ | 54 | 52 | 27, 35 (6), 36 (6) | 7, 8, 9 |
| 17 | $3^{1+2} : 2$ | 54 | 104 | 27, 37, 38 (3) | 8 |
| 18 | $2^3 : S_3$ | 48 | 234 | 32, 29, 33, 40 (3), 48 (4) | 1 |
| 19 | $Q_8 : S_3$ | 48 | 117 | 31, 39 (3), 49 (4) | 3 (2), 10 |
| 20 | $Q_8 \cdot 6$ | 48 | 117 | 31, 41, 47 (4) | 10 |
| 21 | $Q_8 : S_3$ | 48 | 117 | 31, 42 (3), 50 (4) | 10 |
| 22 | $13 : 3$ | 39 | 144 | 46, 74 (13) | 2, 11 |
| 23 | $S_3 \times S_3$ | 36 | 156 | 35 (2), 34, 49 (6) | 7 (2), 14 |
| 24 | $S_3 \times S_3$ | 36 | 312 | 38, 36, 37, 48 (3), 50 (3) | 8 |
| 25 | $3 : S_3 \cdot 2$ | 36 | 78 | 34, 69 (9) | 12, 13, 14 |
| 26 | $D_8 : 2^2$ | 32 | 351 | 40, 43, 44, 45, 41, 42, 39 | 10 |
| 27 | 3^{1+2} | 27 | 52 | 52 (2), 53 (2) | 15 (2), 16, 17 (2) |
| 28 | D_{26} | 26 | 144 | 46, 76 (13) | 11 |
| 29 | $2^2 : S_3$ | 24 | 234 | 51, 56 (3), 64 (4) | 2, 18 |

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| Nr. | Structure | Order | Length | Maximal Subgroups | Minimal Overgroups |
| 30 | D_{24} | 24 | 468 | 50, 47, 49, 57 (3) | 4, 10 |
| 31 | $Q_8 : 3$ | 24 | 117 | 54, 65 (4) | 5 (2), 19, 20, 21 |
| 32 | $A_4 \times 2$ | 24 | 234 | 51, 58, 63 (4) | 18 |
| 33 | $2^2 : S_3$ | 24 | 234 | 51, 57 (3), 66 (4) | 18 |
| 34 | $3 : S_3$ | 18 | 26 | 52, 67 (12) | 15 (4), 23 (6), 25 (3) |
| 35 | $S_3 \times 3$ | 18 | 312 | 52, 67, 65 (3) | 15, 16, 23 |
| 36 | $S_3 \times 3$ | 18 | 312 | 53, 64, 65 (3) | 16, 24 |
| 37 | $3 : S_3$ | 18 | 104 | 53, 66 (9), 68 (3) | 17, 24 (3) |
| 38 | $S_3 \times 3$ | 18 | 312 | 53, 68, 63 (3) | 17, 24 |
| 39 | $Q_8 : 2$ | 16 | 351 | 56, 55, 54 | 6 (2), 19, 26 |
| 40 | $D_8 \times 2$ | 16 | 351 | 56, 59, 57 (2), 58 (2), 60 | 18 (2), 26 |
| 41 | $D_8 : 2$ | 16 | 117 | 59 (3), 61 (3), 54 | 20, 26 (3) |
| 42 | $Q_8 : 2$ | 16 | 351 | 60, 62, 54 | 21, 26 |
| 43 | D_{16} | 16 | 351 | 60, 55, 61 | 26 |
| 44 | $8 : 2$ | 16 | 351 | 59, 62, 55 | 26 |
| 45 | D_{16} | 16 | 351 | 56, 62, 61 | 26 |
| 46 | 13 | 13 | 144 | 78 | 22, 28 |
| 47 | 12 | 12 | 468 | 65, 70 | 9, 20, 30 |
| 48 | D_{12} | 12 | 936 | 66, 63, 64, 71 (3) | 18, 24 |
| 49 | D_{12} | 12 | 468 | 65, 67 (2), 72 (3) | 19, 23 (2), 30 |
| 50 | D_{12} | 12 | 468 | 68 (2), 65, 71 (3) | 21, 24 (2), 30 |
| 51 | A_4 | 12 | 234 | 72, 74 (4) | 29, 32, 33 |
| 52 | 3^2 | 9 | 26 | 75 (4) | 27 (4), 34, 35 (12) |
| 53 | 3^2 | 9 | 104 | 74 (3), 75 | 27, 36 (3), 37, 38 (3) |
| 54 | Q_8 | 8 | 117 | 69 (3) | 12 (2), 31, 39 (3), 41, 42 (3) |
| 55 | 8 | 8 | 351 | 69 | 13 (2), 39, 43, 44 |
| 56 | D_8 | 8 | 351 | 72 (2), 69 | 14 (2), 29 (2), 39, 40, 45 |
| 57 | D_8 | 8 | 702 | 72, 71, 70 | 30 (2), 33, 40 |
| 58 | 2^3 | 8 | 234 | 71 (3), 73 (3), 72 | 32, 40 (3) |
| 59 | 2×4 | 8 | 351 | 73, 69, 70 | 40, 41, 44 |
| 60 | D_8 | 8 | 351 | 71 (2), 69 | 40, 42, 43 |
| 61 | D_8 | 8 | 351 | 73 (2), 69 | 41, 43, 45 |
| 62 | 8 | 8 | 351 | 69 | 42, 44, 45 |
| 63 | 6 | 6 | 936 | 74, 76 | 11 (2), 32, 38, 48 |
| 64 | S_3 | 6 | 312 | 74, 77 (3) | 29 (3), 36, 48 (3) |
| 65 | 6 | 6 | 468 | 75, 77 | 31, 35 (2), 36 (2), 47, 49, 50 |
| 66 | S_3 | 6 | 936 | 74, 76 (3) | 33, 37, 48 |
| 67 | S_3 | 6 | 312 | 75, 77 (3) | 34, 35, 49 (3) |
| 68 | S_3 | 6 | 312 | 75, 76 (3) | 37, 38, 50 (3) |

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| Nr. | Structure | Order | Length | Maximal Subgroups | Minimal Overgroups |
| 69 | 4 | 4 | 351 | 77 | 25 (2), 54, 55, 56, 59, 60, 61, 62 |
| 70 | 4 | 4 | 117 | 77 | 47 (4), 57 (6), 59 (3) |
| 71 | 2^2 | 4 | 702 | 77, 76 (2) | 48 (4), 50 (2), 57, 58, 60 |
| 72 | 2^2 | 4 | 234 | 77 (3) | 49 (6), 51, 56 (3), 57 (3), 58 |
| 73 | 2^2 | 4 | 351 | 76 (2), 77 | 58 (2), 59, 61 (2) |
| 74 | 3 | 3 | 312 | 78 | 22 (6), 51 (3), 53, 63 (3), 64, 66 (3) |
| 75 | 3 | 3 | 52 | 78 | 52 (2), 53 (2), 65 (9), 67 (6), 68 (6) |
| 76 | 2 | 2 | 234 | 78 | 28 (8), 63 (4), 66 (12), 68 (4), 71 (6), 73 (3) |
| 77 | 2 | 2 | 117 | 78 | 64 (8), 65 (4), 67 (8), 69 (3), 70, 71 (6), 72 (6), 73 (3) |
| 78 | 1 | 1 | 1 | | 46 (144), 74 (312), 75 (52), 76 (234), 77 (117) |

TABLE 1. Subgroup lattice of $L_3(3) : 2$

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