

THE SUBGROUP LATTICE OF $L_2(128)$

THOMAS CONNOR AND DIMITRI LEEMANS

Nr.	Structure	Order	Length	Maximal Subgroups	Minimal Overgroups
1	$L_2(128)$	2097024	1	2 (129), 3 (8128), 4 (8256)	
2	$2^7 : 127$	16256	129	6, 7 (128)	1
3	$43 : S_3$	258	8128	5, 8 (3), 218 (43)	1
4	$127 : 2$	254	8256	7, 241 (127)	1
5	129	129	8128	10, 240	3
6	2^7	128	129	9 (127)	2
7	127	127	8256	242	2 (2), 4
8	D_{86}	86	24384	10, 241 (43)	3
9	2^6	64	16383	11 (3), 12 (3), 13 (3), 14 (3), 15 (3), 16 (3), 17 (3), 18 (3), 19 (3), 20 (3), 21 (3), 22 (3), 23 (3), 24 (3), 25 (3), 26 (3), 27 (3), 28 (3), 29 (3), 30 (3), 31 (3)	6
10	43	43	8128	242	5, 8 (3)
11	2^5	32	16383	32, 33, 34, 35, 36, 37 (3), 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60	9 (3)
12	2^5	32	16383	44, 48 (3), 61, 62, 36, 63, 64, 65, 66, 35, 40, 67, 68, 69, 70, 43, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 32	9 (3)
13	2^5	32	16383	73, 60, 83, 56, 84, 79, 85, 86, 71 (3), 37, 41, 87, 88, 89, 90, 81, 53, 76, 91, 92, 72, 42, 93, 94, 95, 96, 97, 98, 32	9 (3)

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Date: July 24, 2013.

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Nr.	Structure	Order	Length	Maximal Subgroups	Minimal Overgroups
14	2^5	32	16383	99, 60, 61, 78, 100, 101, 80, 81, 102, 54, 82, 94, 62, 103 (3), 87, 37, 66, 68, 104, 41, 105, 91, 42, 55, 106, 85, 107, 74, 33	9 (3)
15	2^5	32	16383	98, 83, 48, 96, 108 (3), 85, 100, 109, 110, 34, 69, 74, 104, 80, 106, 40, 59, 50, 111, 36, 73, 95, 56, 57, 112, 101, 84, 87, 33	9 (3)
16	2^5	32	16383	103, 46, 62, 56, 36, 113, 93, 114, 115, 55, 67, 65, 76, 88, 66, 116, 50 (3), 105, 109, 112, 117, 32, 71, 70, 99, 118, 107, 84, 34	9 (3)
17	2^5	32	16383	119, 120, 53, 62, 84, 54, 44, 67, 94, 77, 58, 65 (3), 117, 97, 101, 51, 49, 95, 106, 45, 89, 83, 76, 102, 121, 33, 87, 74, 34	9 (3)
18	2^5	32	16383	32, 116, 90, 68, 106, 79, 64, 96, 54, 118 (3), 100, 89, 112, 95, 63, 69, 37, 107, 122, 77, 39, 94, 121, 53, 55, 102, 108, 70, 35	9 (3)
19	2^5	32	16383	80, 85 (3), 93, 61, 83, 63, 52, 114, 119, 94, 101, 96, 77, 75, 44, 56, 51, 46, 38, 70, 116, 64, 60, 57, 121, 99, 117, 115, 35	9 (3)
20	2^5	32	16383	43, 87, 39, 99, 112, 41, 57, 86 (3), 66, 90, 121, 78, 75, 76, 74, 47, 115, 51, 59, 92, 123, 72, 70, 101, 106, 33, 118, 117, 36	9 (3)
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Nr.	Structure	Order	Length	Maximal Subgroups	Minimal Overgroups
21	2^5	32	16383	111, 120, 74, 63, 98, 99, 82, 92, 84, 41, 35, 65, 100, 64, 105, 110 (3), 40, 88, 115, 77, 70, 89, 117, 42, 123, 39, 60, 113, 37	9 (3)
22	2^5	32	16383	119, 113, 81, 45, 40, 72, 121, 47, 61, 112, 78, 87, 68, 122 (3), 111, 60, 50, 46, 55, 86, 95, 91, 83, 62, 64, 84, 44, 110, 38	9 (3)
23	2^5	32	16383	121, 69, 124, 79, 101, 51 (3), 56, 54, 109, 68, 82, 67, 88, 72, 122, 49, 107, 59, 117, 105, 73, 71, 104, 100, 63, 111, 113, 41, 38	9 (3)
24	2^5	32	16383	58, 123 (3), 81, 85, 83, 68, 78, 45, 96, 79, 102, 61, 90, 69, 124, 56, 49, 50, 104, 36, 99, 92, 53, 63, 62, 34, 113, 120, 39	9 (3)
25	2^5	32	16383	73, 115, 79, 102 (3), 111, 71, 75, 61, 47, 45, 52, 114, 93, 43, 46, 66, 59, 109, 106, 94, 116, 72, 104, 54, 39, 84, 120, 110, 40	9 (3)
26	2^5	32	16383	119 (3), 98, 123, 83, 111, 59, 99, 47, 109, 118, 121, 107, 103, 104, 116, 44, 88, 67, 73, 92, 43, 91, 58, 80, 89, 48, 32, 39, 42	9 (3)
27	2^5	32	16383	80, 124, 113, 53, 98, 75 (3), 122, 88, 45, 58, 55, 94, 108, 105, 69, 34, 117, 114, 66, 72, 42, 92, 95, 89, 103, 57, 64, 49, 43	9 (3)

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Nr.	Structure	Order	Length	Maximal Subgroups	Minimal Overgroups
28	2^5	32	16383	119, 93, 90, 34, 112, 100, 80, 67, 115, 79, 82 (3), 45, 86, 105, 58, 49, 41, 114, 97, 78, 53, 47, 91, 77, 108, 96, 116, 40, 46	9 (3)
29	2^5	32	16383	42, 120, 62, 96, 100, 109, 57, 52 (3), 54, 68, 78, 65, 114, 38, 124, 35, 88, 107, 76, 86, 89, 67, 92, 61, 108, 112, 81, 98, 47	9 (3)
30	2^5	32	16383	43, 87, 106, 116, 101, 69, 44, 74, 32, 46, 110, 60, 49, 115, 124 (3), 81, 38, 113, 33, 90, 114, 107, 93, 97, 91, 118, 98, 120, 48	9 (3)
31	2^5	32	16383	70, 97 (3), 66, 104, 103, 105, 76, 77, 63, 122, 111, 64, 58, 93, 38, 90, 52, 36, 109, 123, 57, 59, 73, 33, 35, 95, 91, 120, 55	9 (3)
32	2^4	16	16383	125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139	11, 12, 13, 16, 18, 26, 30
33	2^4	16	16383	140, 126, 141, 142, 143, 144, 145, 146, 130, 147, 148, 149, 150, 151, 152	11, 14, 15, 17, 20, 30, 31
34	2^4	16	16383	153, 154, 127, 144, 155, 156, 157, 158, 159, 160, 161, 138, 147, 162, 152	11, 15, 16, 17, 24, 27, 28
35	2^4	16	16383	125, 163, 164, 128, 165, 166, 167, 168, 130, 169, 170, 171, 150, 162, 172	11, 12, 18, 19, 21, 29, 31

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Nr.	Structure	Order	Length	Maximal Subgroups	Minimal Overgroups
36	2^4	16	16383	125, 160, 127, 148, 129, 173, 174, 175, 176, 159, 150, 177, 147, 151, 172	11, 12, 15, 16, 20, 24, 31
37	2^4	16	16383	176, 151, 133, 178, 179, 130 (3), 180, 181, 159, 170, 138, 162, 182	11 (3), 13, 14, 18, 21
38	2^4	16	16383	126, 183, 184, 185, 165, 186, 131, 187, 159, 169, 150, 149, 188, 170, 182	11, 19, 22, 23, 29, 30, 31
39	2^4	16	16383	136, 189, 190, 191, 192, 193, 194, 132, 130, 160, 148, 177, 162, 170, 195	11, 18, 20, 21, 24, 25, 26
40	2^4	16	16383	125, 196, 197, 198, 199, 154, 192, 184, 176, 147, 161, 171, 188, 130, 195	11, 12, 15, 21, 22, 25, 28
41	2^4	16	16383	140, 196, 190, 185, 179, 173, 167, 158, 178, 134, 200, 154, 148, 182, 130	11, 13, 14, 20, 21, 23, 28
42	2^4	16	16383	153, 189, 197, 178, 165, 166, 174, 146, 182, 201, 135, 156, 136, 179, 130	11, 13, 14, 21, 26, 27, 29
43	2^4	16	16383	125, 183, 133, 198, 192, 156, 145, 175, 162, 202, 203, 136, 148, 179, 126	11, 12, 20, 25, 26, 27, 30
44	2^4	16	16383	125, 151, 184, 191, 199, 144, 157, 168, 133, 204, 169, 205, 136, 182, 126	11, 12, 17, 19, 22, 26, 30
45	2^4	16	16383	176, 160, 164, 144, 143, 154, 194, 187, 178, 204, 203, 156, 192, 138, 184	11, 17, 22, 24, 25, 27, 28
46	2^4	16	16383	126, 163, 127, 142, 155, 193, 192, 181, 182, 202, 169, 154, 137, 176, 184	11, 16, 19, 22, 25, 28, 30

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Nr.	Structure	Order	Length	Maximal Subgroups	Minimal Overgroups
47	2^4	16	16383	136, 154, 141, 148, 165, 186, 180, 184, 162, 201, 200, 205, 192, 176, 139	11, 20, 22, 25, 26, 28, 29
48	2^4	16	16383	125 (3), 140, 153, 176, 126, 136, 170, 206, 147, 207, 208, 133, 209	11, 12 (3), 15, 26, 30
49	2^4	16	16383	126, 196, 189, 183, 151, 160, 163, 154, 138, 206, 156, 144, 185, 180, 210	11, 17, 23, 24, 27, 28, 30
50	2^4	16	16383	127 (3), 190, 197, 133, 184, 164, 141, 181, 147, 160, 211, 159, 210	11, 15, 16 (3), 22, 24
51	2^4	16	16383	128, 185 (3), 178, 198, 191, 144, 142, 148, 180, 212, 207, 159, 169	11, 17, 19, 20, 23 (3)
52	2^4	16	16383	129, 179, 165 (3), 192, 199, 143, 155, 170, 213, 150, 211, 180, 169	11, 19, 25, 29 (3), 31
53	2^4	16	16383	130, 173, 166, 156, 144, 154, 193, 186, 138, 213, 212, 160, 214, 133, 215	11, 13, 17, 18, 24, 27, 28
54	2^4	16	16383	131, 167, 174, 145, 157, 194, 192, 180, 181, 185, 165, 144, 208, 130, 215	11, 14, 17, 18, 23, 25, 29
55	2^4	16	16383	132, 158, 146, 175, 168, 187, 181, 184, 180, 127, 156, 150, 214, 130, 209	11, 14, 16, 18, 22, 27, 31
56	2^4	16	16383	134, 159, 169, 160, 147, 204, 202, 201, 206, 213, 185, 127, 216, 130, 217	11, 13, 15, 16, 19, 23, 24

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Nr.	Structure	Order	Length	Maximal Subgroups	Minimal Overgroups
57	2^4	16	16383	135, 170, 150, 148, 161, 203, 169, 200, 147, 212, 165, 156, 151, 181, 217	11, 15, 19, 20, 27, 29, 31
58	2^4	16	16383	136, 138, 149, 177, 171, 156, 154, 205, 207, 160, 144, 150, 216, 181, 179	11, 17, 24, 26, 27, 28, 31
59	2^4	16	16383	137, 150, 147, 162, 188, 148, 136, 192, 208, 185, 211, 214, 151, 216, 178	11, 15, 20, 23, 25, 26, 31
60	2^4	16	16383	139, 152, 172, 182, 195, 130, 126, 184, 209, 210, 169, 215, 179, 217, 178	11, 13, 14, 19, 21, 22, 30
61	2^4	16	16383	176, 198, 199, 216, 190, 155, 209, 165, 168, 164, 194, 217, 174, 186, 126	12, 14, 19, 22, 24, 25, 29
62	2^4	16	16383	207, 129, 210, 204, 145, 154, 170, 164, 157, 199, 174, 146, 209, 186, 127	12, 14, 16, 17, 22, 24, 29
63	2^4	16	16383	206, 193, 185, 188, 171, 168, 133, 172, 170, 177, 174, 217, 143, 158, 128	12, 18, 19, 21, 23, 24, 31
64	2^4	16	16383	170, 201, 187, 172, 195, 203, 126, 214, 168, 199, 173, 153, 181, 171, 128	12, 18, 19, 21, 22, 27, 31
65	2^4	16	16383	126, 197, 166, 180, 157 (3), 212, 140, 164, 207, 160, 143, 171, 129	12, 16, 17 (3), 21, 29
66	2^4	16	16383	136, 163, 198, 211, 186, 203, 147, 175, 172, 140, 194, 146, 181, 158, 129	12, 14, 16, 20, 25, 27, 31
67	2^4	16	16383	207, 201, 185, 211, 157, 155, 125, 129, 156, 141, 193, 205, 164, 196, 131	12, 16, 17, 23, 26, 28, 29

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Nr.	Structure	Order	Length	Maximal Subgroups	Minimal Overgroups
68	2^4	16	16383	147, 193, 187, 180, 186, 206, 209, 164, 191, 174, 128, 179, 199, 167, 131	12, 14, 18, 22, 23, 24, 29
69	2^4	16	16383	206, 159, 210, 216, 125, 203, 140, 214, 128, 174, 193, 161, 183, 144, 132	12, 15, 18, 23, 24, 27, 30
70	2^4	16	16383	170, 129, 175, 208, 190, 212, 147, 172, 171, 141, 128, 168, 182, 202, 132	12, 16, 18, 19, 20, 21, 31
71	2^4	16	16383	133, 163, 197, 201, 193, 159, 129, 198, 215, 209, 134 (3), 208, 178	12, 13 (3), 16, 23, 25
72	2^4	16	16383	199, 166, 198, 185, 187, 210, 175, 136, 176, 200, 208, 203, 152, 193, 134	12, 13, 20, 22, 23, 25, 27
73	2^4	16	16383	204, 172, 188, 211, 180, 216, 208, 148, 149, 153, 198, 125, 134, 193, 135	12, 13, 15, 23, 25, 26, 31
74	2^4	16	16383	151, 171, 195, 157, 186, 125, 190, 145, 140, 175, 206, 142, 152, 178, 135	12, 14, 15, 17, 20, 21, 30
75	2^4	16	16383	168, 150, 154, 155, 206, 203 (3), 212, 215, 175, 198, 189, 153, 136	12, 19, 20, 25, 27 (3)
76	2^4	16	16383	157, 165, 147, 129, 164, 214, 172, 175, 149, 200, 134, 142, 189, 207, 133	12, 13, 16, 17, 20, 29, 31
77	2^4	16	16383	191, 170, 168, 157, 172, 128, 171, 156, 140, 215, 176, 149, 213, 196, 137	12, 17, 18, 19, 21, 28, 31
78	2^4	16	16383	169, 177, 199, 164, 140, 174, 141, 192, 209, 175, 153, 200, 186, 196, 138	12, 14, 20, 22, 24, 28, 29

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Nr.	Structure	Order	Length	Maximal Subgroups	Minimal Overgroups
79	2^4	16	16383	184, 174, 173, 207, 194, 193, 128, 162, 208, 198, 206, 134, 213, 196, 138	12, 13, 18, 23, 24, 25, 28
80	2^4	16	16383	205, 217, 153, 160, 146, 161, 168, 179, 125, 203, 208, 142, 186, 196, 137	12, 14, 15, 19, 26, 27, 28
81	2^4	16	16383	182, 209, 174, 143, 181, 164, 199, 183, 125, 134, 152, 189, 213, 186, 139	12, 13, 14, 22, 24, 29, 30
82	2^4	16	16383	144, 186, 176, 171, 158, 196 (3), 167, 202, 153, 193, 178, 207, 139	12, 14, 21, 23, 28 (3)
83	2^4	16	16383	154, 164, 208, 205, 177, 152, 135, 159, 217, 191, 197, 213, 214, 204, 126	13, 15, 17, 19, 22, 24, 26
84	2^4	16	16383	212, 157, 163, 152, 188, 176, 179, 197, 134, 194, 204, 141, 214, 195, 127	13, 15, 16, 17, 21, 22, 25
85	2^4	16	16383	213, 142, 159, 182, 190, 199, 153, 215, 217 (3), 204, 163, 150, 128	13, 14, 15, 19 (3), 24
86	2^4	16	16383	186, 200 (3), 178, 192, 189, 198, 202, 191, 152, 141, 173, 150, 129	13, 20 (3), 22, 28, 29
87	2^4	16	16383	160, 142, 198, 209, 200, 152, 182, 197, 139, 145, 214, 204, 151, 140, 131	13, 14, 15, 17, 20, 22, 30
88	2^4	16	16383	212, 207, 159, 192, 201, 210, 135, 134, 211, 166, 168, 189, 158, 179, 131	13, 16, 21, 23, 26, 27, 29
89	2^4	16	16383	138, 157, 201, 205, 179, 198, 189, 215, 170, 166, 214, 167, 143, 135, 132	13, 17, 18, 21, 26, 27, 29

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Nr.	Structure	Order	Length	Maximal Subgroups	Minimal Overgroups
90	2^4	16	16383	213, 147, 163, 139, 177, 200, 178, 193, 173, 145, 168, 138, 149, 183, 132	13, 18, 20, 24, 28, 30, 31
91	2^4	16	16383	216, 182, 196, 209, 192, 205, 139, 152, 163, 146, 187, 173, 149, 133, 135	13, 14, 22, 26, 28, 30, 31
92	2^4	16	16383	188, 162, 190, 200, 201, 179, 177, 213, 144, 203, 166, 209, 129, 189, 135	13, 20, 21, 24, 26, 27, 29
93	2^4	16	16383	211, 184, 217, 202, 134, 170, 173, 139, 144, 175, 197, 163, 149, 155, 137	13, 16, 19, 25, 28, 30, 31
94	2^4	16	16383	180, 194, 217, 191, 215, 166, 145, 182, 214, 203, 146, 134, 171, 155, 138	13, 14, 17, 18, 19, 25, 27
95	2^4	16	16383	148, 206, 204, 152, 197, 214, 168, 181, 215, 187, 166, 129, 149, 161, 138	13, 15, 17, 18, 22, 27, 31
96	2^4	16	16383	204, 213, 217, 141, 159, 167, 138, 189, 173, 193, 165, 209, 171, 161, 137	13, 15, 18, 19, 24, 28, 29
97	2^4	16	16383	149 (3), 169, 214, 173, 163, 151, 158, 143, 156, 129, 208, 171, 139	13, 17, 28, 30, 31 (3)
98	2^4	16	16383	135, 204, 195, 150, 208, 140, 179, 183, 166, 133, 189, 201, 155, 161, 139	13, 15, 21, 26, 27, 29, 30
99	2^4	16	16383	162, 172, 199, 177, 178, 212, 205, 189, 127, 131, 202, 217, 146, 190, 140	14, 16, 19, 20, 21, 24, 26
100	2^4	16	16383	170, 158, 131, 142, 178, 167, 196, 166, 180, 216, 161, 213, 174, 195, 141	14, 15, 18, 21, 23, 28, 29

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Nr.	Structure	Order	Length	Maximal Subgroups	Minimal Overgroups
101	2^4	16	16383	151, 129, 191, 167, 185, 212, 152, 153, 184, 216, 140, 217, 183, 145, 142	14, 15, 17, 19, 20, 23, 30
102	2^4	16	16383	162, 194 (3), 193, 154, 175, 216, 197, 180, 215, 199, 137, 145, 143	14, 17, 18, 24, 25 (3)
103	2^4	16	16383	151, 158, 164, 216, 179, 175, 139, 146 (3), 181, 215, 153, 131, 190	14 (3), 16, 26, 27, 31
104	2^4	16	16383	177, 208, 167, 194, 199, 216, 139, 142, 156, 146, 188, 159, 211, 206, 148	14, 15, 23, 24, 25, 26, 31
105	2^4	16	16383	212, 167, 196, 155, 210, 175, 181, 160, 146, 158, 188, 128, 201, 178, 149	14, 16, 21, 23, 27, 28, 31
106	2^4	16	16383	202, 140, 161, 152, 194, 215, 164, 145, 180, 188, 173, 136, 132, 142, 151	14, 15, 17, 18, 20, 25, 30
107	2^4	16	16383	146, 169, 174, 183, 137, 180, 207, 131, 133, 211, 167, 201, 132, 190, 152	14, 16, 18, 23, 26, 29, 30
108	2^4	16	16383	161 (3), 211, 160, 145, 150, 153, 206, 135, 213, 167, 176, 214, 141	15 (3), 18, 27, 28, 29
109	2^4	16	16383	188, 216, 127, 211, 175, 208, 148, 201, 165, 207, 191, 161, 163, 131, 143	15, 16, 23, 25, 26, 29, 31
110	2^4	16	16383	195 (3), 162, 210, 176, 188, 168, 202, 135, 197, 140, 167, 182, 143	15, 21 (3), 22, 25, 30

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Nr.	Structure	Order	Length	Maximal Subgroups	Minimal Overgroups
111	2^4	16	16383	189, 158, 128, 195, 211, 208, 205, 145, 169, 187, 216, 176, 188, 197, 148	15, 21, 22, 23, 25, 26, 31
112	2^4	16	16383	155, 213, 161, 207, 197, 167, 186, 132, 192, 172, 187, 127, 141, 200, 151	15, 16, 18, 20, 22, 28, 29
113	2^4	16	16383	210, 164, 212, 201, 187, 158, 137, 198, 182, 177, 206, 127, 183, 195, 144	16, 21, 22, 23, 24, 27, 30
114	2^4	16	16383	184, 141, 158, 165, 207, 203, 135, 215, 202, 199, 137, 160, 183, 163, 155	16, 19, 25, 27, 28, 29, 30
115	2^4	16	16383	190, 184, 196, 143, 200, 212, 136, 163, 172, 202, 145, 159, 137, 195, 155	16, 19, 20, 21, 25, 28, 30
116	2^4	16	16383	184, 210, 155, 148, 213, 202, 137, 163, 171, 205, 132, 133, 131, 194, 153	16, 18, 19, 25, 26, 28, 30
117	2^4	16	16383	158, 150, 185, 157, 142, 212, 203, 202, 190, 172, 191, 210, 201, 197, 138	16, 17, 19, 20, 21, 23, 27
118	2^4	16	16383	131, 206, 173, 137, 145, 166, 172, 132 (3), 181, 191, 133, 141, 162	16, 18 (3), 20, 26, 30
119	2^4	16	16383	191, 166, 156, 204, 188, 192, 205 (3), 207, 137, 126, 139, 190, 153	17, 19, 22, 26 (3), 28
120	2^4	16	16383	128, 166, 183, 149, 202, 143, 157, 177, 165, 194, 211, 151, 209, 195, 154	17, 21, 24, 25, 29, 30, 31
121	2^4	16	16383	205, 174, 204, 211, 215, 212, 171, 191, 185, 187, 132, 126, 200, 142, 162	17, 18, 19, 20, 22, 23, 26

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Nr.	Structure	Order	Length	Maximal Subgroups	Minimal Overgroups
122	2^4	16	16383	139, 141, 174, 198, 187 (3), 188, 210, 154, 181, 214, 191, 168, 169	18, 22 (3), 23, 27, 31
123	2^4	16	16383	156, 177 (3), 186, 204, 189, 147, 190, 128, 216, 173, 143, 162, 210	20, 21, 24 (3), 26, 31
124	2^4	16	16383	144, 183 (3), 142, 136, 210, 155, 135, 209, 206, 131, 143, 169, 173	23, 24, 27, 29, 30 (3)
125	2^3	8	16383	219 (3), 220, 221, 222, 223	32, 35, 36, 40, 43, 44, 48 (3), 67, 69, 73, 74, 80, 81
126	2^3	8	16383	222, 219, 221, 223, 224, 225, 226	32, 33, 38, 43, 44, 46, 48, 49, 60, 61, 64, 65, 83, 119, 121
127	2^3	8	16383	227 (3), 220, 222, 228, 226	32, 34, 36, 46, 50 (3), 55, 56, 62, 84, 99, 109, 112, 113
128	2^3	8	16383	229, 230, 231, 219, 224, 228, 232	32, 35, 51, 63, 64, 68, 69, 70, 77, 79, 85, 105, 111, 120, 123
129	2^3	8	16383	219, 233, 234, 235, 227, 225, 232	32, 36, 52, 62, 65, 66, 67, 70, 71, 76, 86, 92, 95, 97, 101
130	2^3	8	16383	222 (3), 233, 231, 229, 236	32, 33, 35, 37 (3), 39, 40, 41, 42, 53, 54, 55, 56, 60
131	2^3	8	16383	227, 230, 234, 223, 221, 229, 236	32, 38, 54, 67, 68, 87, 88, 99, 100, 103, 107, 109, 116, 118, 124
132	2^3	8	16383	229 (3), 227, 221, 235, 223	32, 39, 55, 69, 70, 89, 90, 106, 107, 112, 116, 118 (3), 121
133	2^3	8	16383	219, 222, 227, 229, 223, 233, 221	32, 37, 43, 44, 48, 50, 53, 63, 71, 76, 91, 98, 107, 116, 118
134	2^3	8	16383	219, 233 (3), 230, 227, 237	32, 41, 56, 71 (3), 72, 73, 76, 79, 81, 84, 88, 93, 94

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<i>continued from previous page</i>					
Nr.	Structure	Order	Length	Maximal Subgroups	Minimal Overgroups
135	2^3	8	16383	220, 231, 234, 221, 223, 238, 233	32, 42, 57, 73, 74, 83, 88, 89, 91, 92, 98, 108, 110, 114, 124
136	2^3	8	16383	219, 222, 223, 235, 237, 238, 221	32, 39, 42, 43, 44, 47, 48, 58, 59, 66, 72, 75, 106, 115, 124
137	2^3	8	16383	221, 224, 227, 229, 223, 237, 239	32, 46, 59, 77, 80, 93, 96, 102, 107, 113, 114, 115, 116, 118, 119
138	2^3	8	16383	222, 228, 225, 229, 233, 238, 239	32, 34, 37, 45, 49, 53, 58, 78, 79, 89, 90, 94, 95, 96, 117
139	2^3	8	16383	223, 226, 232, 236, 221, 233, 239	32, 47, 60, 81, 82, 87, 90, 91, 93, 97, 98, 103, 104, 119, 122
140	2^3	8	16383	231, 219, 235, 225, 220, 223, 236	33, 41, 48, 65, 66, 69, 74, 77, 78, 87, 98, 99, 101, 106, 110
141	2^3	8	16383	229, 227, 235, 239, 234, 220, 226	33, 47, 50, 67, 70, 78, 84, 86, 96, 100, 108, 112, 114, 118, 122
142	2^3	8	16383	236, 235, 230, 225, 224, 220, 223	33, 46, 51, 74, 76, 80, 85, 87, 100, 101, 104, 106, 117, 121, 124
143	2^3	8	16383	231, 228, 237, 232, 234, 225, 223	33, 45, 52, 63, 65, 81, 89, 97, 102, 109, 110, 115, 120, 123, 124
144	2^3	8	16383	222, 228, 230, 239, 238, 223, 225	33, 34, 44, 45, 49, 51, 53, 54, 58, 69, 82, 92, 93, 113, 124
145	2^3	8	16383	229, 235, 237, 223, 220, 236, 225	33, 43, 54, 62, 74, 87, 90, 94, 101, 102, 106, 108, 111, 115, 118
146	2^3	8	16383	236 (3), 227, 221, 232, 238	33, 42, 55, 62, 66, 80, 91, 94, 99, 103 (3), 104, 105, 107
147	2^3	8	16383	219, 228, 235, 227, 232, 222, 220	33, 34, 36, 40, 48, 50, 56, 57, 59, 66, 68, 70, 76, 90, 123
148	2^3	8	16383	235, 230, 237, 221, 222, 232, 220	33, 36, 39, 41, 43, 47, 51, 57, 59, 73, 95, 104, 109, 111, 116

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<i>continued from previous page</i>					
Nr.	Structure	Order	Length	Maximal Subgroups	Minimal Overgroups
149	2^3	8	16383	225, 239, 223, 232 (3), 233	33, 38, 58, 73, 76, 77, 90, 91, 93, 95, 97 (3), 105, 120
150	2^3	8	16383	220, 224, 234, 238, 222, 232, 235	33, 35, 36, 38, 52, 55, 57, 58, 59, 75, 85, 86, 98, 108, 117
151	2^3	8	16383	223, 220, 225, 236, 222, 232, 235	33, 36, 37, 44, 49, 57, 59, 74, 87, 97, 101, 103, 106, 112, 120
152	2^3	8	16383	236, 226, 223, 225, 233, 220, 235	33, 34, 60, 72, 74, 81, 83, 84, 86, 87, 91, 95, 101, 106, 107
153	2^3	8	16383	224, 238, 220, 239, 219, 221, 236	34, 42, 48, 64, 73, 75, 78, 80, 82, 85, 101, 103, 108, 116, 119
154	2^3	8	16383	228, 238, 239, 225, 222, 237, 226	34, 40, 41, 45, 46, 47, 49, 53, 58, 62, 75, 83, 102, 120, 122
155	2^3	8	16383	224, 239, 234, 238, 227, 237, 223	34, 46, 52, 61, 67, 75, 93, 94, 98, 105, 112, 114, 115, 116, 124
156	2^3	8	16383	228, 239, 238, 222, 232, 221, 225	34, 42, 43, 45, 49, 53, 55, 57, 58, 67, 77, 97, 104, 119, 123
157	2^3	8	16383	227, 225 (3), 234, 219, 231	34, 44, 54, 62, 65 (3), 67, 74, 76, 77, 84, 89, 117, 120
158	2^3	8	16383	230, 227, 239, 238, 232, 231, 236	34, 41, 55, 63, 66, 82, 88, 97, 100, 103, 105, 111, 113, 114, 117
159	2^3	8	16383	224, 228, 227, 230, 220, 222, 233	34, 36, 37, 38, 50, 51, 56, 69, 71, 83, 85, 88, 96, 104, 115
160	2^3	8	16383	238, 239, 225, 227, 228, 222, 220	34, 36, 39, 45, 49, 50, 53, 56, 58, 65, 80, 87, 105, 108, 114
161	2^3	8	16383	220 (3), 238, 234, 239, 229	34, 40, 57, 69, 80, 95, 96, 98, 100, 106, 108 (3), 109, 112
162	2^3	8	16383	221, 228, 237, 231, 222, 229, 235	34, 35, 37, 39, 43, 47, 59, 79, 92, 99, 102, 110, 118, 121, 123

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<i>continued from previous page</i>					
Nr.	Structure	Order	Length	Maximal Subgroups	Minimal Overgroups
163	2^3	8	16383	233, 223, 232, 239, 224, 237, 227	35, 46, 49, 66, 71, 84, 85, 90, 91, 93, 97, 109, 114, 115, 116
164	2^3	8	16383	236, 226, 219, 225, 234, 228, 227	35, 45, 50, 61, 62, 65, 67, 68, 76, 78, 81, 83, 103, 106, 113
165	2^3	8	16383	222, 234 (3), 224, 237, 232	35, 38, 42, 47, 52 (3), 54, 57, 61, 76, 96, 109, 114, 120
166	2^3	8	16383	233, 234, 231, 225, 238, 221, 229	35, 42, 53, 65, 72, 88, 89, 92, 94, 95, 98, 100, 118, 119, 120
167	2^3	8	16383	236, 230, 234, 239, 220, 231, 229	35, 41, 54, 68, 82, 89, 96, 100, 101, 104, 105, 107, 108, 110, 112
168	2^3	8	16383	229, 226, 232, 224, 238, 231, 219	35, 44, 55, 61, 63, 64, 70, 75, 77, 80, 88, 90, 95, 110, 122
169	2^3	8	16383	223, 234, 230, 226, 232, 222, 224	35, 38, 44, 46, 51, 52, 56, 57, 60, 78, 97, 107, 111, 122, 124
170	2^3	8	16383	219, 231, 234, 232, 222, 229, 224	35, 37, 38, 39, 48, 52, 57, 62, 63, 64, 70, 77, 89, 93, 100
171	2^3	8	16383	219, 225, 239, 224, 232, 229, 231	35, 40, 58, 63, 64, 65, 70, 74, 77, 82, 94, 96, 97, 116, 121
172	2^3	8	16383	219, 227, 232, 229, 231, 224, 235	35, 36, 60, 63, 64, 66, 70, 73, 76, 77, 99, 112, 115, 117, 118
173	2^3	8	16383	239, 233, 235, 228, 232, 223, 229	36, 41, 53, 64, 79, 86, 90, 91, 93, 96, 97, 106, 118, 123, 124
174	2^3	8	16383	226, 230, 234, 228, 219, 236, 229	36, 42, 54, 61, 62, 63, 68, 69, 78, 79, 81, 100, 107, 121, 122
175	2^3	8	16383	237, 227, 238, 235, 232, 236, 219	36, 43, 55, 66, 70, 72, 74, 75, 76, 78, 93, 102, 103, 105, 109
176	2^3	8	16383	219, 239, 226, 237, 220, 222, 231	36, 37, 40, 45, 46, 47, 48, 61, 72, 77, 82, 84, 108, 110, 111

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<i>continued from previous page</i>					
Nr.	Structure	Order	Length	Maximal Subgroups	Minimal Overgroups
177	2^3	8	16383	221, 228 (3), 235, 232, 231	36, 39, 58, 63, 78, 83, 90, 92, 99, 104, 113, 120, 123 (3)
178	2^3	8	16383	236, 230, 231, 239, 235, 233, 222	37, 41, 42, 45, 51, 59, 60, 71, 74, 82, 86, 90, 99, 100, 105
179	2^3	8	16383	231, 233, 234, 238, 221, 236, 222	37, 41, 42, 43, 52, 58, 60, 68, 80, 84, 88, 89, 92, 98, 103
180	2^3	8	16383	229, 230, 234, 225, 237, 222, 236	37, 47, 49, 51, 52, 54, 55, 65, 68, 73, 94, 100, 102, 106, 107
181	2^3	8	16383	236, 227, 232, 238, 229, 222, 226	37, 46, 50, 54, 55, 57, 58, 64, 66, 81, 95, 103, 105, 118, 122
182	2^3	8	16383	226, 223, 222, 236, 233, 224, 231	37, 38, 41, 42, 44, 46, 60, 70, 81, 85, 87, 91, 94, 110, 113
183	2^3	8	16383	228, 223 (3), 238, 230, 234	38, 43, 49, 69, 81, 90, 98, 101, 107, 113, 114, 120, 124 (3)
184	2^3	8	16383	227, 226, 222, 239, 237, 224, 223	38, 40, 44, 45, 46, 47, 50, 55, 60, 79, 93, 101, 114, 115, 116
185	2^3	8	16383	230 (3), 235, 225, 224, 222	38, 41, 49, 51 (3), 54, 56, 59, 63, 67, 72, 101, 117, 121
186	2^3	8	16383	228, 234, 235, 239, 226, 219, 236	38, 47, 53, 61, 62, 66, 68, 74, 78, 80, 81, 82, 86, 112, 123
187	2^3	8	16383	230, 226 (3), 238, 232, 229	38, 45, 55, 64, 68, 72, 91, 95, 111, 112, 113, 121, 122 (3)
188	2^3	8	16383	221, 230, 237, 226, 220, 232, 231	38, 40, 59, 63, 73, 84, 92, 104, 105, 106, 109, 110, 111, 119, 122
189	2^3	8	16383	233, 238, 221, 228, 235, 234, 231	39, 42, 49, 75, 76, 81, 86, 88, 89, 92, 96, 98, 99, 111, 123
190	2^3	8	16383	236, 227, 235, 228, 221, 224, 231	39, 41, 50, 61, 70, 74, 85, 92, 99, 103, 107, 115, 117, 119, 123

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<i>continued from previous page</i>					
Nr.	Structure	Order	Length	Maximal Subgroups	Minimal Overgroups
191	2^3	8	16383	229, 225, 230, 221, 235, 224, 226	39, 44, 51, 68, 77, 83, 86, 94, 101, 109, 117, 118, 119, 121, 122
192	2^3	8	16383	222, 239, 237, 235, 221, 234, 226	39, 40, 43, 45, 46, 47, 52, 54, 59, 78, 86, 88, 91, 112, 119
193	2^3	8	16383	233, 239, 230, 228, 229, 219, 237	39, 46, 53, 63, 67, 68, 69, 71, 72, 73, 79, 82, 90, 96, 102
194	2^3	8	16383	236, 225, 237 (3), 228, 229	39, 45, 54, 61, 66, 79, 84, 94, 102 (3), 104, 106, 116, 120
195	2^3	8	16383	223, 231 (3), 226, 237, 220	39, 40, 60, 64, 74, 84, 98, 100, 110 (3), 111, 113, 115, 120
196	2^3	8	16383	239 (3), 219, 230, 236, 231	40, 41, 49, 67, 77, 78, 79, 80, 82 (3), 91, 100, 105, 115
197	2^3	8	16383	226, 227, 220, 225, 237, 233, 231	40, 42, 50, 65, 71, 83, 84, 87, 93, 95, 102, 110, 111, 112, 117
198	2^3	8	16383	237, 235, 238, 219, 230, 233, 226	40, 43, 51, 61, 66, 71, 72, 73, 75, 79, 86, 87, 89, 113, 122
199	2^3	8	16383	219, 228, 234, 224, 237, 236, 226	40, 44, 52, 61, 62, 64, 68, 72, 78, 81, 85, 99, 102, 104, 114
200	2^3	8	16383	235 (3), 234, 239, 226, 233	41, 47, 57, 72, 76, 78, 86 (3), 87, 90, 92, 112, 115, 121
201	2^3	8	16383	238, 234, 230, 227, 221, 231, 233	42, 47, 56, 64, 67, 71, 88, 89, 92, 98, 105, 107, 109, 113, 117
202	2^3	8	16383	223, 239, 235, 227, 237, 231, 224	43, 46, 56, 70, 82, 86, 93, 99, 106, 110, 114, 115, 116, 117, 120
203	2^3	8	16383	219, 238 (3), 237, 235, 224	43, 45, 57, 64, 66, 69, 72, 75 (3), 80, 92, 94, 114, 117
204	2^3	8	16383	220, 228, 225, 226, 221, 233, 224	44, 45, 56, 62, 73, 83, 84, 85, 87, 95, 96, 98, 119, 121, 123

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<i>continued from previous page</i>					
Nr.	Structure	Order	Length	Maximal Subgroups	Minimal Overgroups
205	2^3	8	16383	221 (3), 239, 225, 224, 226	44, 47, 58, 67, 80, 83, 89, 91, 99, 111, 116, 119 (3), 121
206	2^3	8	16383	219, 220, 223, 238, 230, 229, 228	48, 49, 56, 63, 68, 69, 74, 75, 79, 95, 104, 108, 113, 118, 124
207	2^3	8	16383	219, 239, 225, 221, 230, 227, 234	48, 51, 58, 62, 65, 67, 76, 79, 82, 88, 107, 109, 112, 114, 119
208	2^3	8	16383	221, 220, 219, 237, 232, 230, 233	48, 54, 59, 70, 71, 72, 73, 79, 80, 83, 97, 98, 104, 109, 111
209	2^3	8	16383	223, 236, 219, 226, 234, 228, 233	48, 55, 60, 61, 62, 68, 71, 78, 81, 87, 91, 92, 96, 120, 124
210	2^3	8	16383	226, 231, 223, 227, 238, 228, 230	49, 50, 60, 62, 69, 72, 88, 105, 110, 113, 116, 117, 122, 123, 124
211	2^3	8	16383	227, 221, 237, 234, 232, 220, 230	50, 52, 59, 66, 67, 73, 88, 93, 104, 107, 108, 109, 111, 120, 121
212	2^3	8	16383	231, 238, 230, 235, 225, 227, 224	51, 53, 57, 65, 70, 75, 84, 88, 99, 101, 105, 113, 115, 117, 121
213	2^3	8	16383	233, 228, 239, 234, 220, 229, 224	52, 53, 56, 77, 79, 81, 83, 85, 90, 92, 96, 100, 108, 112, 116
214	2^3	8	16383	229, 232, 238, 226, 225, 220, 233	53, 55, 59, 64, 69, 76, 83, 84, 87, 89, 94, 95, 97, 108, 122
215	2^3	8	16383	236, 229, 225, 237, 238, 224, 233	53, 54, 60, 71, 75, 77, 85, 89, 94, 95, 102, 103, 106, 114, 121
216	2^3	8	16383	237, 228, 232, 221, 230, 220, 236	56, 58, 59, 61, 69, 73, 91, 100, 101, 102, 103, 104, 109, 111, 123
217	2^3	8	16383	233, 224 (3), 220, 228, 236	56, 57, 60, 61, 63, 80, 83, 85 (3), 93, 94, 96, 99, 101
218	S_3	6	349504	240, 241 (3)	3
219	2^2	4	16383	241 (3)	125 (3), 126, 128, 129, 133, 134, 136, 140, 147, 153, 157, 164, 168, 170, 171, 172, 174, 175, 176, 186, 193, 196, 198, 199, 203, 206, 207, 208, 209

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<i>continued from previous page</i>					
Nr.	Structure	Order	Length	Maximal Subgroups	Minimal Overgroups
220	2^2	4	16383	241 (3)	125, 127, 135, 140, 141, 142, 145, 147, 148, 150, 151, 152, 153, 159, 160, 161 (3), 167, 176, 188, 195, 197, 204, 206, 208, 211, 213, 214, 216, 217
221	2^2	4	16383	241 (3)	125, 126, 131, 132, 133, 135, 136, 137, 139, 146, 148, 153, 156, 162, 166, 177, 179, 188, 189, 190, 191, 192, 201, 204, 205 (3), 207, 208, 211, 216
222	2^2	4	16383	241 (3)	125, 126, 127, 130 (3), 133, 136, 138, 144, 147, 148, 150, 151, 154, 156, 159, 160, 162, 165, 169, 170, 176, 178, 179, 180, 181, 182, 184, 185, 192
223	2^2	4	16383	241 (3)	125, 126, 131, 132, 133, 135, 136, 137, 139, 140, 142, 143, 144, 145, 149, 151, 152, 155, 163, 169, 173, 182, 183 (3), 184, 195, 202, 206, 209, 210
224	2^2	4	16383	241 (3)	126, 128, 137, 142, 150, 153, 155, 159, 163, 165, 168, 169, 170, 171, 172, 182, 184, 185, 190, 191, 199, 202, 203, 204, 205, 212, 213, 215, 217 (3)
					<i>continued on next page</i>

<i>continued from previous page</i>					
Nr.	Structure	Order	Length	Maximal Subgroups	Minimal Overgroups
225	2^2	4	16383	241 (3)	126, 129, 138, 140, 142, 143, 144, 145, 149, 151, 152, 154, 156, 157 (3), 160, 164, 166, 171, 180, 185, 191, 194, 197, 204, 205, 207, 212, 214, 215
226	2^2	4	16383	241 (3)	126, 127, 139, 141, 152, 154, 164, 168, 169, 174, 176, 181, 182, 184, 186, 187 (3), 188, 191, 192, 195, 197, 198, 199, 200, 204, 205, 209, 210, 214
227	2^2	4	16383	241 (3)	127 (3), 129, 131, 132, 133, 134, 137, 141, 146, 147, 155, 157, 158, 159, 160, 163, 164, 172, 175, 181, 184, 190, 197, 201, 202, 207, 210, 211, 212
228	2^2	4	16383	241 (3)	127, 128, 138, 143, 144, 147, 154, 156, 159, 160, 162, 164, 173, 174, 177 (3), 183, 186, 189, 190, 193, 194, 199, 204, 206, 209, 210, 213, 216, 217
229	2^2	4	16383	241 (3)	128, 130, 131, 132 (3), 133, 137, 138, 141, 145, 161, 162, 166, 167, 168, 170, 171, 172, 173, 174, 180, 181, 187, 191, 193, 194, 206, 213, 214, 215
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<i>continued from previous page</i>					
Nr.	Structure	Order	Length	Maximal Subgroups	Minimal Overgroups
230	2^2	4	16383	241 (3)	128, 131, 134, 142, 144, 148, 158, 159, 167, 169, 174, 178, 180, 183, 185 (3), 187, 188, 191, 193, 196, 198, 201, 206, 207, 208, 210, 211, 212, 216
231	2^2	4	16383	241 (3)	128, 130, 135, 140, 143, 157, 158, 162, 166, 167, 168, 170, 171, 172, 176, 177, 178, 179, 182, 188, 189, 190, 195 (3), 196, 197, 201, 202, 210, 212
232	2^2	4	16383	241 (3)	128, 129, 139, 143, 146, 147, 148, 149 (3), 150, 151, 156, 158, 163, 165, 168, 169, 170, 171, 172, 173, 175, 177, 181, 187, 188, 208, 211, 214, 216
233	2^2	4	16383	241 (3)	129, 130, 133, 134 (3), 135, 138, 139, 149, 152, 159, 163, 166, 173, 178, 179, 182, 189, 193, 197, 198, 200, 201, 204, 208, 209, 213, 214, 215, 217
234	2^2	4	16383	241 (3)	129, 131, 135, 141, 143, 150, 155, 157, 161, 164, 165 (3), 166, 167, 169, 170, 174, 179, 180, 183, 186, 189, 192, 199, 200, 201, 207, 209, 211, 213
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<i>continued from previous page</i>					
Nr.	Structure	Order	Length	Maximal Subgroups	Minimal Overgroups
235	2^2	4	16383	241 (3)	129, 132, 136, 140, 141, 142, 145, 147, 148, 150, 151, 152, 162, 172, 173, 175, 177, 178, 185, 186, 189, 190, 191, 192, 198, 200 (3), 202, 203, 212
236	2^2	4	16383	241 (3)	130, 131, 139, 140, 142, 145, 146 (3), 151, 152, 153, 158, 164, 167, 174, 175, 178, 179, 180, 181, 182, 186, 190, 194, 196, 199, 209, 215, 216, 217
237	2^2	4	16383	241 (3)	134, 136, 137, 143, 145, 148, 154, 155, 162, 163, 165, 175, 176, 180, 184, 188, 192, 193, 194 (3), 195, 197, 198, 199, 202, 203, 208, 211, 215, 216
238	2^2	4	16383	241 (3)	135, 136, 138, 144, 146, 150, 153, 154, 155, 156, 158, 160, 161, 166, 168, 175, 179, 181, 183, 187, 189, 198, 201, 203 (3), 206, 210, 212, 214, 215
239	2^2	4	16383	241 (3)	137, 138, 139, 141, 144, 149, 153, 154, 155, 156, 158, 160, 161, 163, 167, 171, 173, 176, 178, 184, 186, 192, 193, 196 (3), 200, 202, 205, 207, 213
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Nr.	Structure	Order	Length	Maximal Subgroups	Minimal Overgroups
240	3	3	8128	242	5, 218 (43)
241	2	2	16383	242	4 (64), 8 (64), 218 (64), 219 (3), 220 (3), 221 (3), 222 (3), 223 (3), 224 (3), 225 (3), 226 (3), 227 (3), 228 (3), 229 (3), 230 (3), 231 (3), 232 (3), 233 (3), 234 (3), 235 (3), 236 (3), 237 (3), 238 (3), 239 (3)
242	1	1	1		7 (8256), 10 (8128), 240 (8128), 241 (16383)

TABLE 1. Subgroup lattice of $L_2(128)$

UNIVERSITÉ LIBRE DE BRUXELLES, DÉPARTEMENT DE MATHÉMATIQUES - C.P.216, BOULEVARD DU TRIOMPHE,
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