THE SUBGROUP LATTICE OF $P\Sigma L_3(4) \cong L_3(4) : 2$

THOMAS CONNOR AND DIMITRI LEEMANS

Nr.	Structure	Order	Length	Maximal Subgroups	Minimal Overgroups
1	$L_3(4) : 2$	40320	1	2, 3 (21), 4 (21), 7 (56), 11 (120), 21 (280)	
2	$L_3(4)$	20160	1	5 (21), 6 (21), 9 (112), 10 (56), 17 (120), 18 (240), 35 (280)	1
3	$2^4 : A_5 : 2$	1920	21	5, 8 (5), 12 (6), 14 (10), 23 (16), 24 (16)	1
4	$2^4 : A_5 : 2$	1920	21	6, 8 (5), 13 (6), 15 (10), 24 (16), 25 (16)	1
5	$2^4 : A_5$	960	21	16 (5), 19 (6), 26 (10), 45 (16), 46 (16), 47 (32)	2, 3
6	$2^4 : A_5$	960	21	16 (5), 20 (6), 27 (10), 45 (16), 48 (32), 49 (16)	2, 4
7	S_6	720	56	10, 23 (6), 25 (6), 36 (10), 50 (15), 51 (15)	1
8	$2^3 \cdot 2 : 2 : 2 : S_3$	384	105	16, 22 (3), 28 (4), 29 (4), 30 (4)	3, 4
9	A_6	360	112	48 (6), 47 (6), 57 (10), 74 (15), 75 (15)	2
10	A_6	360	56	46 (6), 49 (6), 58 (10), 76 (15), 77 (15)	2, 7
11	$L_2(7) \times 2$	336	120	17, 50 (7), 51 (7), 56 (8)	1
12	$2^4 : 5 : 2 \cdot 2$	320	126	19, 38 (5), 84 (16)	3
13	$2^4 : 5 : 2 \cdot 2$	320	126	20, 39 (5), 84 (16)	4
14	$2 \times 4 : 2 : 2 : S_3$	192	210	31, 28, 26, 40 (3), 50 (4)	3
15	$2^3 \cdot 2 : 2 : S_3$	192	210	29, 27, 32, 41 (3), 51 (4)	4

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Nr.	Structure	Order	Length	Maximal Subgroups	Minimal Overgroups
16	$2 \times 4 : 2 : 2 : 2 : 3$	192	105	42, 52 (8), 53 (4), 54 (4), 55 (4)	5, 6, 8
17	$L_2(7)$	168	120	77 (7), 76 (7), 83 (8)	2, 11
18	$L_2(7)$	168	240	74 (7), 75 (7), 83 (8)	2
19	$2^4 : 5 : 2$	160	126	33, 60 (5), 115 (16)	5, 12
20	$2^4 : 5 : 2$	160	126	34, 61 (5), 115 (16)	6, 13
21	$3 : S_3 \cdot 2 \cdot 2 : 2$	144	280	35, 37, 36, 87 (9)	1
22	$2^{1+4} \cdot 2^2$	128	315	40, 39, 38, 41, 42, 43, 44	8
23	S_5	120	336	46, 78 (5), 84 (6), 109 (10)	3, 7
24	S_5	120	336	45, 79 (5), 84 (6), 109 (10)	3, 4
25	S_5	120	336	49, 80 (5), 84 (6), 109 (10)	4, 7
26	$2^4 : S_3$	96	210	54, 60 (3), 76 (4), 75 (8)	5, 14
27	$2^4 : S_3$	96	210	53, 61 (3), 74 (8), 77 (4)	6, 15
28	$2^4 : S_3$	96	210	54, 62 (3), 80 (4), 79 (8)	8 (2), 14
29	$2^4 : S_3$	96	210	53, 63 (3), 78 (4), 79 (8)	8 (2), 15
30	$4^2 : S_3$	96	420	55, 64 (3), 79 (4)	8
31	$2^4 : 3 : 2$	96	210	54, 62, 81 (4)	14
32	$2^4 : 3 : 2$	96	210	53, 63, 82 (4)	15
33	$2^4 : 5$	80	126	88, 139 (16)	19
34	$2^4 : 5$	80	126	89, 139 (16)	20
35	M_9	72	280	58, 57 (2), 117 (9)	2, 21
36	$S_3 \times S_3 : 2$	72	280	58, 59 (2), 118 (9)	7 (2), 21
37	$3 : S_3 \cdot 2 \cdot 2$	72	280	58, 119 (9)	21
38	$2 \times 4 : 2 : 2 : 2$	64	315	60, 65, 66	12 (2), 22
39	$2^3 \cdot 2 : 2 \cdot 2$	64	315	67, 65, 61	13 (2), 22
40	$2^{1+4} : 2$	64	315	62 (2), 60, 68 (2), 67, 69	14 (2), 22
41	$2^{1+4} : 2$	64	315	69, 70 (2), 66, 61, 63 (2)	15 (2), 22
42	$2 \times 4 : 2 : 2 : 2$	64	105	60 (3), 71 (6), 72 (3), 61 (3)	16, 22 (3)
43	$2 \times 4 : 2 : 2 \cdot 2$	64	315	67, 66, 72	22
44	$2^{1+4} \cdot 2$	64	315	69, 64 (2), 65, 72, 73 (2)	22

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Nr.	Structure	Order	Length	Maximal Subgroups	Minimal Overgroups
45	A_5	60	336	110 (5), 115 (6), 136 (10)	5, 6, 24
46	A_5	60	336	111 (5), 115 (6), 136 (10)	5, 10, 23
47	A_5	60	672	112 (5), 115 (6), 136 (10)	5, 9
48	A_5	60	672	113 (5), 115 (6), 136 (10)	6, 9
49	A_5	60	336	114 (5), 115 (6), 136 (10)	6, 10, 25
50	$2^3 : S_3$	48	840	76, 81, 80, 90 (3), 109 (4)	7, 11, 14
51	$2^3 : S_3$	48	840	77, 78, 82, 90 (3), 109 (4)	7, 11, 15
52	$4^2 : 3$	48	840	91, 110 (4)	16
53	$2^4 : 3$	48	210	89, 110 (8), 112 (8), 111 (4)	16 (2), 27, 29, 32
54	$2^4 : 3$	48	210	88, 113 (8), 110 (8), 114 (4)	16 (2), 26, 28, 31
55	$4^2 : 3$	48	420	92, 110 (4)	16, 30
56	$7 : 3 \times 2$	42	960	83, 108, 137 (7)	11
57	$3 : S_3 \cdot 2$	36	560	85, 140 (9)	9 (2), 35
58	$3 : S_3 \cdot 2$	36	280	85, 141 (9)	10 (2), 35, 36, 37
59	$S_3 \times S_3$	36	560	86 (2), 85, 109 (6)	36
60	$2 \times 4 : 2 : 2$	32	315	88, 93 (2), 94, 95 (2), 96	19 (2), 26 (2), 38, 40, 42
61	$2^3 \cdot 2 : 2$	32	315	93 (2), 97, 94, 98 (2), 89	20 (2), 27 (2), 39, 41, 42
62	$D_8 \times 2 : 2$	32	210	88, 99 (3), 100 (3)	28 (3), 31, 40 (3)
63	$2^3 \cdot 2 : 2$	32	210	101 (3), 102 (3), 89	29 (3), 32, 41 (3)
64	$D_8 : 4$	32	630	103, 104, 92	30 (2), 44
65	$8 : 2 : 2$	32	315	94, 104 (2)	38, 39, 44
66	$2 \times 4 : 2 : 2$	32	315	94, 102 (2)	38, 41, 43
67	$D_8 \times 2 \cdot 2$	32	315	99 (2), 94	39, 40, 43
68	$2 \times 4 : 2 : 2$	32	630	100, 99, 96	40
69	2^{1+4}	32	315	90 (4), 101 (2), 103 (2), 105 (4), 100 (2), 94	40, 41, 44
70	$2 \times 4 : 2 : 2$	32	630	101, 102, 97	41
71	$2 \times 4 : 2 : 2$	32	630	93, 106, 95, 97, 96, 91, 98	42
72	$2 \times 4 : 2 : 2$	32	315	94, 92, 95 (2), 98 (2), 106	42, 43, 44

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Nr.	Structure	Order	Length	Maximal Subgroups	Minimal Overgroups
73	$Q_8 \cdot 2^2$	32	630	87 (2), 107 (2), 103, 104, 106	44
74	$2^2 : S_3$	24	1680	112, 120 (3), 136 (4)	9, 18, 27
75	$2^2 : S_3$	24	1680	113, 120 (3), 136 (4)	9, 18, 26
76	$2^2 : S_3$	24	840	114, 121 (3), 136 (4)	10, 17, 26, 50
77	$2^2 : S_3$	24	840	111, 121 (3), 136 (4)	10, 17, 27, 51
78	$2^2 : S_3$	24	840	111, 122 (3), 138 (4)	23 (2), 29, 51
79	$2^2 : S_3$	24	1680	110, 123 (3), 138 (4)	24, 28, 29, 30
80	$2^2 : S_3$	24	840	114, 124 (3), 138 (4)	25 (2), 28, 50
81	$A_4 \times 2$	24	840	114, 125, 137 (4)	31, 50
82	$A_4 \times 2$	24	840	111, 126, 137 (4)	32, 51
83	$7 : 3$	21	960	135, 149 (7)	17, 18 (2), 56
84	$D_{10} \cdot 2$	20	2016	115, 142 (5)	12, 13, 23, 24, 25
85	$3 : S_3$	18	280	116, 136 (12)	57 (2), 58, 59 (2)
86	$S_3 \times 3$	18	1120	116, 138, 137 (3)	59
87	$D_8 \cdot 2$	16	1260	118, 117, 119	21 (2), 73
88	2^4	16	21	127 (15)	33 (6), 54 (10), 60 (15), 62 (10)
89	2^4	16	21	128 (15)	34 (6), 53 (10), 61 (15), 63 (10)
90	$D_8 \times 2$	16	1260	118, 129, 124, 125, 126, 121, 122	50 (2), 51 (2), 69
91	4^2	16	210	130 (3)	52 (4), 71 (3)
92	4^2	16	105	131 (3)	55 (4), 64 (6), 72 (3)
93	$D_8 \times 2$	16	630	120 (4), 127, 128, 130	60, 61, 71
94	$D_8 \times 2$	16	315	121 (4), 127, 131, 128	60, 61, 65, 66, 67, 69, 72
95	$2 \times 4 : 2$	16	630	127, 131, 130	60, 71, 72
96	$2 \times 4 : 2$	16	315	127, 130 (2)	60, 68 (2), 71 (2)
97	$2 \times 4 : 2$	16	315	128, 130 (2)	61, 70 (2), 71 (2)
98	$2 \times 4 : 2$	16	630	128, 131, 130	61, 71, 72
99	$2 \times 4 : 2$	16	630	127, 132 (2)	62, 67, 68
100	$D_8 \times 2$	16	630	124 (2), 123 (2), 127, 132, 125	62, 68, 69
101	$D_8 \times 2$	16	630	122 (2), 123 (2), 126, 133, 128	63, 69, 70
102	$2 \times 4 : 2$	16	630	128, 133 (2)	63, 66, 70
103	$D_8 : 2$	16	630	118, 134, 129 (2), 123 (2), 131	64, 69, 73
104	$8 : 2$	16	630	131, 119 (2)	64, 65, 73
105	$D_8 : 2$	16	1260	132, 133, 121, 134, 124, 129, 122	69

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Nr.	Structure	Order	Length	Maximal Subgroups	Minimal Overgroups
106	$Q_8 \times 2$	16	315	117 (4), 130 (2), 131	71 (2), 72, 73 (2)
107	Q_{16}	16	1260	134, 119, 117	73
108	14	14	960	135, 150	56
109	D_{12}	12	3360	137, 138, 136, 143 (3)	23, 24, 25, 50, 51, 59
110	A_4	12	1680	144, 149 (4)	45, 52 (2), 53, 54, 55, 79
111	A_4	12	840	145, 149 (4)	46 (2), 53, 77, 78, 82
112	A_4	12	1680	146, 149 (4)	47 (2), 53, 74
113	A_4	12	1680	147, 149 (4)	48 (2), 54, 75
114	A_4	12	840	148, 149 (4)	49 (2), 54, 76, 80, 81
115	D_{10}	10	2016	139, 151 (5)	19, 20, 45, 46, 47 (2), 48 (2), 49, 84
116	3^2	9	280	149 (4)	85, 86 (4)
117	Q_8	8	1260	140 (2), 141	35 (2), 87, 106, 107
118	D_8	8	630	143 (2), 141	36 (4), 87 (2), 90 (2), 103
119	8	8	1260	141	37 (2), 87, 104, 107
120	D_8	8	2520	147, 140, 146	74 (2), 75 (2), 93
121	D_8	8	1260	148, 141, 145	76 (2), 77 (2), 90, 94, 105
122	D_8	8	1260	143, 142, 145	78 (2), 90, 101, 105
123	D_8	8	1260	143, 142, 144	79 (4), 100, 101, 103
124	D_8	8	1260	148, 142, 143	80 (2), 90, 100, 105
125	2^3	8	210	143 (6), 148	81 (4), 90 (6), 100 (3)
126	2^3	8	210	143 (6), 145	82 (4), 90 (6), 101 (3)
127	2^3	8	315	144, 148 (2), 147 (4)	88, 93 (2), 94, 95 (2), 96, 99 (2), 100 (2)
128	2^3	8	315	145 (2), 146 (4), 144	89, 93 (2), 94, 97, 98 (2), 101 (2), 102 (2)
129	2×4	8	1260	143, 141, 142	90, 103, 105
130	2×4	8	630	140 (2), 144	91, 93, 95, 96, 97, 98, 106
131	2×4	8	315	141 (2), 144	92, 94, 95 (2), 98 (2), 103 (2), 104 (2), 106
132	2×4	8	630	142 (2), 148	99 (2), 100, 105 (2)
133	2×4	8	630	142 (2), 145	101, 102 (2), 105 (2)
134	Q_8	8	630	142 (2), 141	103, 105 (2), 107 (2)
135	7	7	960	152	83, 108
136	S_3	6	3360	149, 151 (3)	45, 46, 47 (2), 48 (2), 49, 74 (2), 75 (2), 76, 77, 85, 109

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Nr.	Structure	Order	Length	Maximal Subgroups	Minimal Overgroups
137	6	6	3360	149, 150	56 (2), 81, 82, 86, 109
138	S_3	6	1120	149, 150 (3)	78 (3), 79 (6), 80 (3), 86, 109 (3)
139	5	5	2016	152	33, 34, 115
140	4	4	1260	151	57 (4), 117 (2), 120 (2), 130
141	4	4	630	151	58 (4), 117 (2), 118, 119 (2), 121 (2), 129 (2), 131, 134
142	4	4	1260	151	84 (8), 122, 123, 124, 129, 132, 133, 134
143	2^2	4	1260	151, 150 (2)	109 (8), 118, 122, 123, 124, 125, 126, 129
144	2^2	4	105	151 (3)	110 (16), 123 (12), 127 (3), 128 (3), 130 (6), 131 (3)
145	2^2	4	210	151 (3)	111 (4), 121 (6), 122 (6), 126, 128 (3), 133 (3)
146	2^2	4	420	151 (3)	112 (4), 120 (6), 128 (3)
147	2^2	4	420	151 (3)	113 (4), 120 (6), 127 (3)
148	2^2	4	210	151 (3)	114 (4), 121 (6), 124 (6), 125, 127 (3), 132 (3)
149	3	3	1120	152	83 (6), 110 (6), 111 (3), 112 (6), 113 (6), 114 (3), 116, 136 (3), 137 (3), 138
150	2	2	120	152	108 (8), 137 (28), 138 (28), 143 (21)
151	2	2	315	152	115 (32), 136 (32), 140 (4), 141 (2), 142 (4), 143 (4), 144, 145 (2), 146 (4), 147 (4), 148 (2)
152	1	1	1		135 (960), 139 (2016), 149 (1120), 150 (120), 151 (315)

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Nr.	Structure	Order	Length	Maximal Subgroups	Minimal Overgroups

TABLE 1. Subgroup lattice of $L_3(4) : 2$

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