

THE SUBGROUP LATTICE OF $P\Sigma L(2,9) \cong S_6$

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Nr.	Structure	Order	Length	Maximal Subgroups	Minimal Overgroups
1	S_6	720	1	2, 3 (6), 4 (6), 5 (10), 8 (15), 9 (15)	
2	A_6	360	1	6 (6), 7 (6), 10 (10), 13 (15), 14 (15)	1
3	S_5	120	6	6, 15 (5), 19 (6), 24 (10)	1
4	S_5	120	6	7, 16 (5), 19 (6), 25 (10)	1
5	$S_3 \times S_3 : 2$	72	10	10, 11, 12, 30 (9)	1
6	A_5	60	6	26 (5), 28 (6), 37 (10)	2, 3
7	A_5	60	6	27 (5), 28 (6), 38 (10)	2, 4
8	$2^3 : S_3$	48	15	16, 17, 13, 23 (3), 25 (4)	1
9	$2^3 : S_3$	48	15	15, 18, 14, 23 (3), 24 (4)	1
10	$3 : S_3 \cdot 2$	36	10	20, 44 (9)	2, 5
11	$S_3 \times S_3$	36	10	20, 21 (2), 25 (6)	5
12	$S_3 \times S_3$	36	10	20, 22 (2), 24 (6)	5
13	S_4	24	15	27, 31 (3), 38 (4)	2, 8
14	S_4	24	15	26, 31 (3), 37 (4)	2, 9
15	S_4	24	15	26, 32 (3), 39 (4)	3 (2), 9
16	S_4	24	15	27, 33 (3), 40 (4)	4 (2), 8
17	$A_4 \times 2$	24	15	27, 34, 41 (4)	8
18	$A_4 \times 2$	24	15	26, 35, 42 (4)	9
19	$D_{10} \cdot 2$	20	36	28, 45 (5)	3, 4
20	$3 : S_3$	18	10	29, 38 (6), 37 (6)	10, 11, 12
21	$S_3 \times 3$	18	20	29, 40, 41 (3)	11
22	$S_3 \times 3$	18	20	29, 39, 42 (3)	12
23	$D_8 \times 2$	16	45	30, 36, 32, 35, 34, 31, 33	8, 9
24	D_{12}	12	60	37, 42, 39, 46 (3)	3, 9, 12
25	D_{12}	12	60	38, 41, 40, 47 (3)	4, 8, 11
26	A_4	12	15	48, 51 (4)	6 (2), 14, 15, 18
27	A_4	12	15	49, 52 (4)	7 (2), 13, 16, 17

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Nr.	Structure	Order	Length	Maximal Subgroups	Minimal Overgroups
28	D_{10}	10	36	43, 53 (5)	6, 7, 19
29	3^2	9	10	52 (2), 51 (2)	20, 21 (2), 22 (2)
30	D_8	8	45	47, 44, 46	5 (2), 23
31	D_8	8	45	48, 44, 49	13, 14, 23
32	D_8	8	45	46, 45, 48	15, 23
33	D_8	8	45	49, 45, 47	16, 23
34	2^3	8	15	46 (3), 50 (3), 49	17, 23 (3)
35	2^3	8	15	47 (3), 50 (3), 48	18, 23 (3)
36	2×4	8	45	44, 45, 50	23
37	S_3	6	60	51, 53 (3)	6, 14, 20, 24
38	S_3	6	60	52, 53 (3)	7, 13, 20, 25
39	S_3	6	20	51, 54 (3)	15 (3), 22, 24 (3)
40	S_3	6	20	52, 55 (3)	16 (3), 21, 25 (3)
41	6	6	60	52, 55	17, 21, 25
42	6	6	60	51, 54	18, 22, 24
43	5	5	36	56	28
44	4	4	45	53	10 (2), 30, 31, 36
45	4	4	45	53	19 (4), 32, 33, 36
46	2^2	4	45	54 (2), 53	24 (4), 30, 32, 34
47	2^2	4	45	55 (2), 53	25 (4), 30, 33, 35
48	2^2	4	15	53 (3)	26, 31 (3), 32 (3), 35
49	2^2	4	15	53 (3)	27, 31 (3), 33 (3), 34
50	2^2	4	45	54, 55, 53	34, 35, 36
51	3	3	20	56	26 (3), 29, 37 (3), 39, 42 (3)
52	3	3	20	56	27 (3), 29, 38 (3), 40, 41 (3)
53	2	2	45	56	28 (4), 37 (4), 38 (4), 44, 45, 46, 47, 48, 49, 50
54	2	2	15	56	39 (4), 42 (4), 46 (6), 50 (3)
55	2	2	15	56	40 (4), 41 (4), 47 (6), 50 (3)
56	1	1	1		43 (36), 51 (20), 52 (20), 53 (45), 54 (15), 55 (15)

TABLE 1. Subgroup lattice of S_6

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