

THE SUBGROUP LATTICE OF $PGU_3(5) \cong U_3(5) : 3$

THOMAS CONNOR AND DIMITRI LEEMANS

Nr.	Structure	Order	Length	Maximal Subgroups	Minimal Overgroups
1	$U_3(5) : 3$	378000	1	2, 3 (126), 8 (525), 16 (1750), 17 (1750), 30 (6000)	
2	$U_3(5)$	126000	1	4 (150), 6 (126), 9 (525), 15 (525)	1
3	$5^{1+2} : 24$	3000	126	5, 6, 21 (25)	1
4	A_7	2520	150	12 (7), 18 (30), 22 (21), 26 (35)	2
5	$5^{1+2} : 12$	1500	126	7, 10, 31 (25)	3
6	$5^{1+2} : 8$	1000	126	10, 37 (25)	2, 3
7	$5^{1+2} : 6$	750	126	11, 14, 44 (25)	5
8	$2 \cdot A_5 \times 3 : 2$	720	525	13, 15, 19 (5), 21 (6), 27 (10)	1
9	M_{10}	720	525	12, 28 (10), 58 (36), 63 (45)	2
10	$5^{1+2} : 4$	500	126	14, 25 (30)	5, 6
11	$5^{1+2} : 3$	375	126	20, 64 (25)	7
12	A_6	360	525	32 (12), 38 (10), 47 (30)	4 (2), 9
13	$2 \cdot A_5 \times 3$	360	525	23, 29 (5), 31 (6), 39 (10)	8
14	$5^{1+2} : 2$	250	126	20, 34 (30)	7, 10
15	$2 \cdot A_5 : 2$	240	525	23, 35 (5), 37 (6), 48 (10)	2, 8
16	$M_9 : 3$	216	1750	28, 33 (4), 49 (9)	1
17	$6^2 : S_3$	216	1750	24, 26, 27 (3), 33 (4)	1
18	$L_2(7)$	168	2250	47 (14), 54 (8)	4 (2)
19	$Q_8 \times 3 : S_3$	144	2625	29, 35, 36 (3), 40 (4)	8
20	5^{1+2}	125	126	46 (6)	11, 14
21	$30 \cdot 2 \cdot 2$	120	3150	31, 37, 50 (5)	3, 8
22	$A_5 : 2$	120	3150	32, 47 (5), 58 (6), 65 (10)	4
23	$2 \cdot A_5$	120	525	51 (5), 59 (6), 66 (10)	13, 15
24	$A_4 \times 3 : 3$	108	1750	41 (2), 42, 43, 45 (4)	17

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Nr.	Structure	Order	Length	Maximal Subgroups	Minimal Overgroups
25	$5^2 : 2 \cdot 2$	100	3780	34, 58 (5), 59 (5)	10
26	$A_4 : S_3$	72	1750	42, 47 (9), 48 (3), 60 (4)	4 (3), 17
27	$2 \times 6 : S_3$	72	5250	40, 43, 39, 48, 52 (3)	8, 17
28	M_9	72	1750	38 (3), 77 (9)	9 (3), 16
29	$Q_8 : 3^2$	72	2625	51, 53, 49 (2), 61 (4)	13, 19
30	$7 : 3 \times 3$	63	6000	55, 56, 57, 54, 74 (7)	1
31	$30 \cdot 2$	60	3150	44, 59, 67 (5)	5, 13, 21
32	A_5	60	3150	68 (5), 72 (6), 81 (10)	12 (2), 22
33	$3^{1+2} : 2$	54	7000	45, 60, 62 (3)	16, 17
34	$5^2 : 2$	50	3780	46, 72, 73 (5)	14, 25
35	$Q_8 : S_3$	48	2625	51, 63 (3), 65 (4)	15, 19
36	$Q_8 : 6$	48	7875	50, 53, 52, 63	19
37	$Q_{20} \cdot 2$	40	3150	59, 78 (5)	6, 15, 21
38	$3 : S_3 \cdot 2$	36	5250	60, 87 (9)	12, 28
39	$6 \cdot S_3$	36	5250	61, 66, 67 (3)	13, 27
40	$S_3 \times 6$	36	5250	61, 62 (2), 65, 69 (3)	19 (2), 27
41	$A_4 \times 3$	36	3500	70 (3), 71, 74 (4)	24
42	$A_4 \times 3$	36	1750	68 (3), 71, 75 (4)	24, 26
43	6^2	36	1750	61 (3), 71, 69 (3)	24, 27 (3)
44	30	30	3150	64, 73, 82	7, 31
45	3^{1+2}	27	7000	76, 74 (2), 75	24, 33
46	5^2	25	756	85 (5), 86	20, 34 (5)
47	$2^2 : S_3$	24	15750	68, 79 (3), 81 (4)	12, 18 (2), 22, 26
48	$2^2 : S_3$	24	5250	66, 71, 65, 79 (3)	15, 26, 27
49	$Q_8 : 3$	24	5250	77, 83 (4)	16 (3), 29
50	24	24	7875	67, 78	21 (2), 36
51	$Q_8 : 3$	24	2625	77, 84 (4)	23, 29, 35
52	$D_8 \times 3$	24	7875	67, 69 (2), 79	27 (2), 36
53	$Q_8 \times 3$	24	2625	67 (3), 77	29, 36 (3)
54	$7 : 3$	21	6000	80, 89 (7)	18 (3), 30
55	21	21	6000	80, 90	30
56	$7 : 3$	21	6000	80, 90 (7)	30
57	$7 : 3$	21	6000	80, 90 (7)	30
58	$D_{10} \cdot 2$	20	18900	72, 87 (5)	9, 22, 25
59	Q_{20}	20	3150	73, 87 (5)	23, 25 (6), 31, 37
60	$3 : S_3$	18	1750	75, 81 (12)	26 (4), 33 (4), 38 (3)
61	3×6	18	5250	76, 83 (2), 84, 82	29 (2), 39, 40, 43
62	$S_3 \times 3$	18	10500	76, 81, 83 (3)	33 (2), 40
63	$Q_8 : 2$	16	7875	79, 78, 77	9 (3), 35, 36
64	15	15	3150	86, 91	11, 44
65	D_{12}	12	5250	81 (2), 84, 88 (3)	22 (6), 35 (2), 40, 48
66	Q_{12}	12	5250	84, 87 (3)	23, 39, 48

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Nr.	Structure	Order	Length	Maximal Subgroups	Minimal Overgroups
67	12	12	7875	82, 87	31 (2), 39 (2), 50, 52, 53
68	A_4	12	5250	88, 89 (4)	32 (3), 42, 47 (3)
69	2×6	12	5250	83 (2), 82, 88	40 (3), 43, 52 (3)
70	A_4	12	10500	88, 90 (4)	41
71	2×6	12	1750	84 (3), 88	41 (2), 42, 43, 48 (3)
72	D_{10}	10	3780	85, 92 (5)	32 (5), 34, 58 (5)
73	10	10	3150	86, 92	34 (6), 44, 59
74	3^2	9	14000	89, 90 (3)	30 (3), 41, 45
75	3^2	9	1750	89 (4)	42 (4), 45 (4), 60
76	3^2	9	1750	91 (3), 89	45 (4), 61 (3), 62 (6)
77	Q_8	8	2625	87 (3)	28 (6), 49 (2), 51, 53, 63 (3)
78	8	8	7875	87	37 (2), 50, 63
79	D_8	8	7875	88 (2), 87	47 (6), 48 (2), 52, 63
80	7	7	6000	93	54, 55, 56, 57
81	S_3	6	10500	89, 92 (3)	32 (3), 47 (6), 60 (2), 62, 65
82	6	6	525	91, 92	44 (6), 61 (10), 67 (15), 69 (10)
83	6	6	10500	91, 92	49 (2), 61, 62 (3), 69
84	6	6	5250	89, 92	51 (2), 61, 65, 66, 71
85	5	5	3780	93	46, 72
86	5	5	126	93	46 (6), 64 (25), 73 (25)
87	4	4	7875	92	38 (6), 58 (12), 59 (2), 66 (2), 67, 77, 78, 79
88	2^2	4	1750	92 (3)	65 (9), 68 (3), 69 (3), 70 (6), 71, 79 (9)
89	3	3	1750	93	54 (24), 68 (12), 74 (8), 75 (4), 76, 81 (6), 84 (3)
90	3	3	6000	93	55, 56 (7), 57 (7), 70 (7), 74 (7)
91	3	3	525	93	64 (6), 76 (10), 82, 83 (20)
92	2	2	525	93	72 (36), 73 (6), 81 (60), 82, 83 (20), 84 (10), 87 (15), 88 (10)
93	1	1	1		80 (6000), 85 (3780), 86 (126), 89 (1750), 90 (6000), 91 (525), 92 (525)

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TABLE 1. Subgroup lattice of $U_3(5) : 3$

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