

## THE SUBGROUP LATTICE OF $PGL_3(4) \cong L_3(4) : 3$

THOMAS CONNOR AND DIMITRI LEEMANS

| Nr. | Structure                    | Order | Length | Maximal Subgroups                            | Minimal Overgroups |
|-----|------------------------------|-------|--------|--|--------------------|
| 1   | $L_3(4) : 3$                 | 60480 | 1      | 2, 3 (21), 4 (21), 15 (280), 34 (960)        |                    |
| 2   | $L_3(4)$                     | 20160 | 1      | 5 (21), 6 (21), 10 (168), 21 (360), 32 (280) | 1                  |
| 3   | $2^4 : A_5 : 3$              | 2880  | 21     | 5, 7 (5), 8 (6), 11 (10), 20 (16)            | 1                  |
| 4   | $2^4 : A_5 : 3$              | 2880  | 21     | 6, 7 (5), 9 (6), 12 (10), 20 (16)            | 1                  |
| 5   | $2^4 : A_5$                  | 960   | 21     | 16 (5), 22 (6), 26 (10), 35 (48), 36 (16)    | 2, 3               |
| 6   | $2^4 : A_5$                  | 960   | 21     | 16 (5), 23 (6), 27 (10), 37 (48), 36 (16)    | 2, 4               |
| 7   | $2^4 : 3 : 2 : 2 : 3$        | 576   | 105    | 17, 18, 19, 16, 24 (4), 25 (4)               | 3, 4               |
| 8   | $2^4 : 5 : 3 : 2$            | 480   | 126    | 13, 22, 28 (5), 52 (16)                      | 3                  |
| 9   | $2^4 : 5 : 3 : 2$            | 480   | 126    | 14, 23, 29 (5), 52 (16)                      | 4                  |
| 10  | $A_6$                        | 360   | 168    | 35 (6), 37 (6), 47 (10), 54 (15), 55 (15)    | 2                  |
| 11  | $2^4 : 3 : S_3$              | 288   | 210    | 25, 26, 28 (3), 63 (16)                      | 3                  |
| 12  | $2^4 : 3 : S_3$              | 288   | 210    | 24, 27, 29 (3), 63 (16)                      | 4                  |
| 13  | $2^4 : 5 : 3$                | 240   | 126    | 30, 39, 72 (16)                              | 8                  |
| 14  | $2^4 : 5 : 3$                | 240   | 126    | 31, 40, 72 (16)                              | 9                  |
| 15  | $M_9 : 3$                    | 216   | 280    | 32, 38 (4), 56 (9)                           | 1                  |
| 16  | $4^2 : 2 : 2 : 3$            | 192   | 105    | 33, 41 (12), 42 (4), 43 (4)                  | 5, 6, 7            |
| 17  | $2^4 : 3 : 2 : 2$            | 192   | 105    | 29 (3), 33, 44 (4)                           | 7                  |
| 18  | $2 \times 4 : 2 : 2 : 2 : 3$ | 192   | 105    | 33, 45 (4), 46 (12), 44 (4)                  | 7                  |
| 19  | $2^4 : 3 : 2 : 2$            | 192   | 105    | 28 (3), 33, 45 (4)                           | 7                  |
| 20  | $A_5 \times 3$               | 180   | 336    | 36, 48 (5), 52 (6), 63 (10)                  | 3, 4               |
| 21  | $L_2(7)$                     | 168   | 360    | 55 (7), 54 (7), 59 (8)                       | 2                  |
| 22  | $2^4 : 5 : 2$                | 160   | 126    | 30, 49 (5), 79 (16)                          | 5, 8               |

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| Nr.                                 | Structure                   | Order | Length | Maximal Subgroups          | Minimal Overgroups       |
| 23                                  | $2^4 : 5 : 2$               | 160   | 126    | 31, 50 (5), 79 (16)        | 6, 9                     |
| 24                                  | $A_4 \times 2 \times 2 : 3$ | 144   | 210    | 40, 45 (2), 43, 48 (8)     | 7 (2), 12                |
| 25                                  | $A_4 \times 2 \times 2 : 3$ | 144   | 210    | 44 (2), 39, 42, 48 (8)     | 7 (2), 11                |
| 26                                  | $2^4 : S_3$                 | 96    | 210    | 42, 49 (3), 54 (12)        | 5, 11                    |
| 27                                  | $2^4 : S_3$                 | 96    | 210    | 43, 50 (3), 55 (12)        | 6, 12                    |
| 28                                  | $2^4 : 3 : 2$               | 96    | 315    | 39, 49, 57 (4)             | 8 (2), 11 (2), 19        |
| 29                                  | $2^4 : 3 : 2$               | 96    | 315    | 40, 50, 58 (4)             | 9 (2), 12 (2), 17        |
| 30                                  | $2^4 : 5$                   | 80    | 126    | 65, 91 (16)                | 13, 22                   |
| 31                                  | $2^4 : 5$                   | 80    | 126    | 66, 91 (16)                | 14, 23                   |
| 32                                  | $M_9$                       | 72    | 280    | 47 (3), 83 (9)             | 2, 15                    |
| 33                                  | $4^2 : 2 : 2$               | 64    | 105    | 49 (3), 51 (9), 50 (3)     | 16, 17, 18, 19           |
| 34                                  | $7 : 3 \times 3$            | 63    | 960    | 60, 61, 62, 59, 80 (7)     | 1                        |
| 35                                  | $A_5$                       | 60    | 1008   | 73 (5), 79 (6), 89 (10)    | 5, 10                    |
| 36                                  | $A_5$                       | 60    | 336    | 74 (5), 79 (6), 89 (10)    | 5, 6, 20                 |
| 37                                  | $A_5$                       | 60    | 1008   | 75 (5), 79 (6), 89 (10)    | 6, 10                    |
| 38                                  | $3^{1+2} : 2$               | 54    | 1120   | 53, 64, 63 (3)             | 15                       |
| 39                                  | $2^4 : 3$                   | 48    | 21     | 65, 76 (20)                | 13 (6), 25 (10), 28 (15) |
| 40                                  | $2^4 : 3$                   | 48    | 21     | 66, 77 (20)                | 14 (6), 24 (10), 29 (15) |
| 41                                  | $4^2 : 3$                   | 48    | 1260   | 67, 74 (4)                 | 16                       |
| 42                                  | $2^4 : 3$                   | 48    | 210    | 65, 75 (12), 74 (8)        | 16 (2), 25, 26           |
| 43                                  | $2^4 : 3$                   | 48    | 210    | 66, 74 (8), 73 (12)        | 16 (2), 24, 27           |
| 44                                  | $A_4 \times 2 \times 2$     | 48    | 420    | 58 (3), 65, 78 (4)         | 17, 18, 25               |
| 45                                  | $A_4 \times 2 \times 2$     | 48    | 420    | 57 (3), 66, 78 (4)         | 18, 19, 24               |
| 46                                  | $Q_8 : 6$                   | 48    | 1260   | 56, 68, 78 (4)             | 18                       |
| 47                                  | $3 : S_3 \cdot 2$           | 36    | 840    | 64, 92 (9)                 | 10 (2), 32               |
| 48                                  | $A_4 \times 3$              | 36    | 1680   | 77, 78, 74, 76, 81 (4)     | 20, 24, 25               |
| 49                                  | $D_8 \times 2 : 2$          | 32    | 315    | 65, 69 (3), 70 (3)         | 22 (2), 26 (2), 28, 33   |
| 50                                  | $2^3 \cdot 2 : 2$           | 32    | 315    | 70 (3), 71 (3), 66         | 23 (2), 27 (2), 29, 33   |
| 51                                  | $2 \times 4 : 2 : 2$        | 32    | 945    | 69 (2), 71 (2), 70, 67, 68 | 33                       |
| 52                                  | $15 : 2$                    | 30    | 2016   | 72, 79, 90 (5)             | 8, 9, 20                 |
| 53                                  | $3^{1+2}$                   | 27    | 1120   | 81, 80 (2), 82             | 38                       |
| 54                                  | $2^2 : S_3$                 | 24    | 2520   | 75, 84 (3), 89 (4)         | 10, 21, 26               |
| 55                                  | $2^2 : S_3$                 | 24    | 2520   | 73, 84 (3), 89 (4)         | 10, 21, 27               |
| 56                                  | $Q_8 : 3$                   | 24    | 1260   | 83, 90 (4)                 | 15 (2), 46               |
| 57                                  | $A_4 \times 2$              | 24    | 1260   | 76, 85, 90 (4)             | 28, 45                   |
| 58                                  | $A_4 \times 2$              | 24    | 1260   | 77, 86, 90 (4)             | 29, 44                   |
| 59                                  | $7 : 3$                     | 21    | 960    | 88, 96 (7)                 | 21 (3), 34               |
| 60                                  | $7 : 3$                     | 21    | 960    | 88, 97 (7)                 | 34                       |
| 61                                  | $7 : 3$                     | 21    | 960    | 88, 97 (7)                 | 34                       |
| 62                                  | 21                          | 21    | 960    | 88, 97                     | 34                       |

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| Nr.                                 | Structure        | Order | Length | Maximal Subgroups  | Minimal Overgroups                                       |
| 63                                  | $S_3 \times 3$   | 18    | 3360   | 81, 89, 90 (3)     | 11, 12, 20, 38   |
| 64                                  | $3 : S_3$        | 18    | 280    | 82, 89 (12)        | 38 (4), 47 (3)   |
| 65                                  | $2^4$            | 16    | 21     | 86 (15)            | 30 (6), 39, 42 (10), 44 (20), 49 (15)                    |
| 66                                  | $2^4$            | 16    | 21     | 85 (15)            | 31 (6), 40, 43 (10), 45 (20), 50 (15)                    |
| 67                                  | $4^2$            | 16    | 315    | 87 (3)             | 41 (4), 51 (3)   |
| 68                                  | $Q_8 \times 2$   | 16    | 315    | 83 (4), 87 (3)     | 46 (4), 51 (3)   |
| 69                                  | $2 \times 4 : 2$ | 16    | 945    | 86, 87 (2)         | 49, 51 (2)   |
| 70                                  | $D_8 \times 2$   | 16    | 945    | 84 (4), 86, 87, 85 | 49, 50, 51   |
| 71                                  | $2 \times 4 : 2$ | 16    | 945    | 85, 87 (2)         | 50, 51 (2)   |
| 72                                  | 15               | 15    | 2016   | 91, 98             | 13, 14, 52   |
| 73                                  | $A_4$            | 12    | 2520   | 93, 96 (4)         | 35 (2), 43, 55   |
| 74                                  | $A_4$            | 12    | 1680   | 94, 96 (4)         | 36, 41 (3), 42, 43, 48                                   |
| 75                                  | $A_4$            | 12    | 2520   | 95, 96 (4)         | 37 (2), 42, 54   |
| 76                                  | $A_4$            | 12    | 420    | 94, 98 (4)         | 39, 48 (4), 57 (3)                                       |
| 77                                  | $A_4$            | 12    | 420    | 94, 98 (4)         | 40, 48 (4), 58 (3)                                       |
| 78                                  | $2 \times 6$     | 12    | 1680   | 90 (3), 94         | 44, 45, 46 (3), 48                                       |
| 79                                  | $D_{10}$         | 10    | 2016   | 91, 99 (5)         | 22, 23, 35 (3), 36, 37 (3), 52                           |
| 80                                  | $3^2$            | 9     | 2240   | 96, 97 (3)         | 34 (3), 53   |
| 81                                  | $3^2$            | 9     | 1120   | 98 (3), 96         | 48 (6), 53, 63 (3)                                       |
| 82                                  | $3^2$            | 9     | 280    | 96 (4)             | 53 (4), 64   |
| 83                                  | $Q_8$            | 8     | 1260   | 92 (3)             | 32 (2), 56, 68   |
| 84                                  | $D_8$            | 8     | 3780   | 95, 92, 93         | 54 (2), 55 (2), 70                                       |
| 85                                  | $2^3$            | 8     | 315    | 93 (6), 94         | 57 (4), 66, 70 (3), 71 (3)                               |
| 86                                  | $2^3$            | 8     | 315    | 95 (6), 94         | 58 (4), 65, 69 (3), 70 (3)                               |
| 87                                  | $2 \times 4$     | 8     | 945    | 92 (2), 94         | 67, 68, 69 (2), 70, 71 (2)                               |
| 88                                  | 7                | 7     | 960    | 100                | 59, 60, 61, 62   |
| 89                                  | $S_3$            | 6     | 3360   | 96, 99 (3)         | 35 (3), 36, 37 (3), 54 (3), 55 (3), 63, 64               |
| 90                                  | 6                | 6     | 5040   | 98, 99             | 52 (2), 56, 57, 58, 63 (2), 78                           |
| 91                                  | 5                | 5     | 2016   | 100                | 30, 31, 72, 79   |
| 92                                  | 4                | 4     | 1890   | 99                 | 47 (4), 83 (2), 84 (2), 87                               |
| 93                                  | $2^2$            | 4     | 630    | 99 (3)             | 73 (4), 84 (6), 85 (3)                                   |
| 94                                  | $2^2$            | 4     | 105    | 99 (3)             | 74 (16), 76 (4), 77 (4), 78 (16), 85 (3), 86 (3), 87 (9) |

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| Nr.                                 | Structure | Order | Length | Maximal Subgroups | Minimal Overgroups   |
| 95                                  | $2^2$     | 4     | 630    | 99 (3)            | 75 (4), 84 (6), 86 (3)   |
| 96                                  | 3         | 3     | 1120   | 100               | 59 (6), 73 (9), 74 (6),<br>75 (9), 80 (2), 81, 82,<br>89 (3)       |
| 97                                  | 3         | 3     | 960    | 100               | 60 (7), 61 (7), 62, 80<br>(7)                                      |
| 98                                  | 3         | 3     | 336    | 100               | 72 (6), 76 (5), 77 (5),<br>81 (10), 90 (15)                        |
| 99                                  | 2         | 2     | 315    | 100               | 79 (32), 89 (32), 90<br>(16), 92 (6), 93 (6),<br>94, 95 (6)        |
| 100                                 | 1         | 1     | 1      |                   | 88 (960), 91 (2016),<br>96 (1120), 97 (960),<br>98 (336), 99 (315) |

TABLE 1. Subgroup lattice of  $L_3(4) : 3$ 

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