

THE SUBGROUP LATTICE OF $PFU_3(3) \cong U_3(3) : 2$

THOMAS CONNOR AND DIMITRI LEEMANS

Nr.	Structure	Order	Length	Maximal Subgroups	Minimal Overgroups
1	$U_3(3) : 2$	12096	1	2, 3 (28), 4 (36), 8 (63), 9 (63)	
2	$U_3(3)$	6048	1	5 (28), 10 (36), 14 (63), 15 (63)	1
3	$3^{1+2} : (D_8 \cdot 2)$	432	28	6, 5, 7, 23 (9)	1
4	$L_2(7) : 2$	336	36	10, 29 (8), 50 (21), 64 (28)	1
5	$3^{1+2} : 8$	216	28	11, 39 (9)	2, 3
6	$3^{1+2} : D_8$	216	28	12 (2), 11, 40 (9)	3
7	$3^{1+2} : Q_8$	216	28	13 (2), 11, 41 (9)	3
8	$2^{1+4} : S_3$	192	63	16, 17, 15, 20 (3), 23 (4)	1
9	$D_8 \times 2 : 2 : S_3$	192	63	14, 18, 19, 20 (3), 24 (4)	1
10	$L_2(7)$	168	36	42 (14), 46 (8)	2, 4
11	$3^{1+2} : 4$	108	28	21, 65 (9)	5, 6, 7
12	$3^{1+2} : 2^2$	108	56	22 (2), 21, 30 (6)	6
13	$3^{1+2} : 4$	108	56	21, 66 (9)	7
14	$4^2 : S_3$	96	63	25, 31 (3), 42 (4)	2, 9
15	$D_8 : 2 \cdot S_3$	96	63	26, 31 (3), 39 (4)	2, 8
16	$D_8 : 2 : S_3$	96	63	27 (2), 26, 32 (3), 40 (4)	8
17	$2^{1+4} : 3$	96	63	26, 28 (2), 33, 41 (4)	8
18	$4^2 : 3 : 2$	96	63	25, 34, 43 (4)	9
19	$4^2 : S_3$	96	63	25, 35 (3), 44 (4)	9
20	$2^{1+4} : 2$	64	189	34, 35, 36, 37, 33, 31, 32	8, 9
21	$3^{1+2} : 2$	54	28	38, 47 (12)	11, 12 (2), 13 (2)
22	$3^{1+2} : 2$	54	112	38, 48, 49 (3)	12
23	$Q_8 : S_3$	48	252	39, 41, 40, 51 (3)	3, 8
24	$2^3 : S_3$	48	252	43, 42, 44, 52 (3), 64 (4)	9
25	$4^2 : 3$	48	63	53, 67 (4)	14, 18, 19
26	$Q_8 \cdot 6$	48	63	45, 54, 65 (4)	15, 16, 17

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Nr.	Structure	Order	Length	Maximal Subgroups	Minimal Overgroups
27	$Q_8 : S_3$	48	126	45, 55 (3), 68 (4)	16
28	$Q_8 \cdot 6$	48	126	45, 56, 66 (4)	17
29	$7 : 3 : 2$	42	288	46, 63, 85 (7)	4
30	$S_3 \times S_3$	36	336	49, 47, 48, 64 (3), 68 (3)	12
31	$Q_8 : 4$	32	189	53, 54, 57	14, 15, 20
32	$D_8 : 2^2$	32	189	58, 50 (2), 59, 54, 55 (2)	16, 20
33	2^{1+4}	32	63	60 (3), 61 (3), 52 (6), 56 (2), 54	17, 20 (3)
34	$D_8 \times 2 : 2$	32	63	60 (3), 58 (3), 53	18, 20 (3)
35	$Q_8 : 4$	32	189	53, 59, 61	19, 20
36	$8 : 2 : 2$	32	189	60, 57, 59	20
37	$D_8 : 2^2$	32	189	58, 62 (2), 57, 61, 51 (2)	20
38	3^{1+2}	27	28	69 (4)	21, 22 (4)
39	$4 \cdot S_3$	24	252	65, 70 (3)	5, 15, 23
40	D_{24}	24	252	68 (2), 65, 71 (3)	6, 16, 23
41	$Q_8 \times 3$	24	252	66 (2), 65, 72	7, 17, 23
42	$2^2 : S_3$	24	252	67, 73 (3), 86 (4)	10 (2), 14, 24
43	$A_4 \times 2$	24	252	67, 74, 85 (4)	18, 24
44	$2^2 : S_3$	24	252	67, 75 (3), 87 (4)	19, 24
45	$Q_8 : 3$	24	63	76, 88 (4)	26, 27 (2), 28 (2)
46	$7 : 3$	21	288	84, 96 (7)	10, 29
47	$S_3 \times 3$	18	336	69, 86, 88 (3)	21, 30
48	$3 : S_3$	18	112	69, 87 (9), 89 (3)	22, 30 (3)
49	$S_3 \times 3$	18	336	69, 89, 85 (3)	22, 30
50	D_{16}	16	378	77, 78, 73	4 (2), 32
51	$Q_8 : 2$	16	378	71, 70, 72	23 (2), 37
52	$D_8 \times 2$	16	378	75 (2), 79, 73, 74, 80, 81	24 (2), 33
53	4^2	16	63	82 (3)	25, 31 (3), 34, 35 (3)
54	$D_8 : 2$	16	63	73 (3), 82 (3), 76	26, 31 (3), 32 (3), 33
55	$Q_8 : 2$	16	378	77, 78, 76	27, 32
56	$D_8 : 2$	16	126	79 (3), 80 (3), 76	28, 33
57	$8 : 2$	16	189	82, 70 (2)	31, 36, 37
58	$D_8 \times 2$	16	189	81 (2), 71 (2), 77 (2), 82	32, 34, 37
59	$8 : 2$	16	189	82, 78 (2)	32, 35, 36
60	$D_8 \times 2$	16	189	79 (2), 83 (2), 74, 82, 81	33, 34, 36
61	$D_8 : 2$	16	189	75 (2), 72, 83, 80 (2), 82	33, 35, 37

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Nr.	Structure	Order	Length	Maximal Subgroups	Minimal Overgroups
62	D_{16}	16	378	71, 70, 83	37
63	D_{14}	14	288	84, 98 (7)	29
64	D_{12}	12	1008	86, 85, 87, 90 (3)	4, 24, 30
65	12	12	252	88, 91	11, 26, 39, 40, 41
66	12	12	504	88, 92	13, 28, 41
67	A_4	12	252	93, 96 (4)	25, 42, 43, 44
68	D_{12}	12	504	88, 89 (2), 90 (3)	27, 30 (2), 40
69	3^2	9	112	96 (3), 97	38, 47 (3), 48, 49 (3)
70	8	8	378	91	39 (2), 51, 57, 62
71	D_8	8	378	90 (2), 91	40 (2), 51, 58, 62
72	Q_8	8	63	92 (2), 91	41 (4), 51 (6), 61 (3)
73	D_8	8	189	93 (2), 94	42 (4), 50 (2), 52 (2), 54
74	2^3	8	63	95 (6), 93	43 (4), 52 (6), 60 (3)
75	D_8	8	378	95, 92, 93	44 (2), 52 (2), 61
76	Q_8	8	63	94 (3)	45, 54, 55 (6), 56 (2)
77	D_8	8	378	90 (2), 94	50, 55, 58
78	8	8	378	94	50, 55, 59
79	D_8	8	378	95 (2), 94	52, 56, 60
80	2×4	8	378	92, 94, 95	52, 56, 61
81	2^3	8	189	90 (4), 93, 95 (2)	52 (2), 58 (2), 60
82	2×4	8	189	91, 94, 93	53, 54, 57, 58, 59, 60, 61
83	D_8	8	189	95 (2), 91	60 (2), 61, 62 (2)
84	7	7	288	100	46, 63
85	6	6	1008	96, 98	29 (2), 43, 49, 64
86	S_3	6	336	96, 99 (3)	42 (3), 47, 64 (3)
87	S_3	6	1008	96, 98 (3)	44, 48, 64
88	6	6	252	97, 99	45, 47 (4), 65, 66 (2), 68 (2)
89	S_3	6	336	97, 98 (3)	48, 49, 68 (3)
90	2^2	4	756	98 (2), 99	64 (4), 68 (2), 71, 77, 81
91	4	4	63	99	65 (4), 70 (6), 71 (6), 72, 82 (3), 83 (3)
92	4	4	126	99	66 (4), 72, 75 (3), 80 (3)
93	2^2	4	63	99 (3)	67 (4), 73 (6), 74, 75 (6), 81 (3), 82 (3)
94	4	4	189	99	73, 76, 77 (2), 78 (2), 79 (2), 80 (2), 82
95	2^2	4	378	98 (2), 99	74, 75, 79 (2), 80, 81, 83

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Nr.	Structure	Order	Length	Maximal Subgroups	Minimal Overgroups
96	3	3	336	100	46 (6), 67 (3), 69, 85 (3), 86, 87 (3)
97	3	3	28	100	69 (4), 88 (9), 89 (12)
98	2	2	252	100	63 (8), 85 (4), 87 (12), 89 (4), 90 (6), 95 (3)
99	2	2	63	100	86 (16), 88 (4), 90 (12), 91, 92 (2), 93 (3), 94 (3), 95 (6)
100	1	1	1		84 (288), 96 (336), 97 (28), 98 (252), 99 (63)

TABLE 1. Subgroup lattice of $U_3(3) : 2$

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