

## THE SUBGROUP LATTICE OF $M_{23}$

THOMAS CONNOR AND DIMITRI LEEMANS

| Nr. | Structure          | Order    | Length | Maximal Subgroups  | Minimal Overgroups |
|-----|--------------------|----------|--------|--|--------------------|
| 1   | $M_{23}$           | 10200960 | 1      | 2 (23), 3 (253), 4 (253), 5 (506), 7 (1288), 8 (1771), 49 (40320)          |                    |
| 2   | $M_{22}$           | 443520   | 23     | 6 (22), 9 (77), 13 (176), 14 (176), 15 (231), 17 (330), 24 (616), 26 (672) | 1                  |
| 3   | $2^4 : A_7$        | 40320    | 253    | 9 (7), 11 (15), 12 (15), 13 (16), 16 (21), 19 (35)                         | 1                  |
| 4   | $L_3(4) : 2$       | 40320    | 253    | 6, 16 (21), 15 (21), 25 (56), 40 (120), 71 (280)                           | 1                  |
| 5   | $A_8$              | 20160    | 506    | 14 (8), 18 (15), 17 (15), 25 (28), 27 (35), 36 (56)                        | 1                  |
| 6   | $L_3(4)$           | 20160    | 253    | 20 (21), 21 (21), 37 (112), 38 (56), 64 (240), 65 (120), 93 (280)          | 2 (2), 4           |
| 7   | $M_{11}$           | 7920     | 1288   | 24 (11), 26 (12), 71 (55), 75 (66), 112 (165)                              | 1                  |
| 8   | $2^4 : A_5 : S_3$  | 5760     | 1771   | 10, 15 (3), 19 (5), 22 (6), 27 (10), 36 (16)                               | 1                  |
| 9   | $2^4 : A_6$        | 5760     | 1771   | 23 (6), 20 (6), 28 (10), 31 (15), 32 (15), 39 (16), 37 (16)                | 2, 3               |
| 10  | $2^4 : A_5 : 3$    | 2880     | 1771   | 21, 29 (5), 30 (6), 45 (10), 63 (16)                                       | 8                  |
| 11  | $2^3 : L_2(7) : 2$ | 2688     | 3795   | 18, 33 (7), 32 (7), 41 (8)   | 3                  |

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| Nr.                                 | Structure                             | Order | Length | Maximal Subgroups   | Minimal Overgroups |
| 12                                  | $2^4 : L_2(7)$                        | 2688  | 3795   | 32 (7), 34 (7), 42 (8),<br>40 (8), 41 (8)                   | 3                  |
| 13                                  | $A_7$                                 | 2520  | 4048   | 37 (7), 65 (15), 66<br>(15), 76 (21), 94 (35)               | 2, 3               |
| 14                                  | $A_7$                                 | 2520  | 4048   | 38 (7), 66 (15), 64<br>(15), 77 (21), 94 (35)               | 2, 5               |
| 15                                  | $2^4 : A_5 : 2$                       | 1920  | 5313   | 21, 31 (5), 43 (6), 51<br>(10), 76 (16), 77 (16)            | 2, 4, 8            |
| 16                                  | $2^4 : A_5 : 2$                       | 1920  | 5313   | 20, 31 (5), 44 (6), 52<br>(10), 75 (16), 76 (16)            | 3, 4               |
| 17                                  | $2^3 : L_2(7)$                        | 1344  | 7590   | 53 (7), 51 (7), 66 (8),<br>64 (8), 67 (8)                   | 2, 5               |
| 18                                  | $2^3 : L_2(7)$                        | 1344  | 3795   | 54 (7), 53 (7), 66<br>(16), 68 (8)                          | 5 (2), 11          |
| 19                                  | $2^3 \cdot 2 : 2 : 2 : 3 : S_3$       | 1152  | 8855   | 29, 31 (3), 35 (3), 33<br>(3), 34 (3), 46 (4), 47<br>(4)    | 3, 8               |
| 20                                  | $2^4 : A_5$                           | 960   | 5313   | 55 (5), 69 (6), 79<br>(10), 104 (16), 105<br>(32), 106 (16) | 6, 9 (2), 16       |
| 21                                  | $2^4 : A_5$                           | 960   | 1771   | 55 (5), 70 (6), 80<br>(10), 107 (16), 106<br>(48)           | 6 (3), 10, 15 (3)  |
| 22                                  | $2^4 : 5 : 2 \cdot S_3$               | 960   | 10626  | 30, 43 (3), 56 (5), 108<br>(16)                             | 8                  |
| 23                                  | $2^4 : A_5$                           | 960   | 10626  | 57 (5), 69 (6), 81<br>(10), 105 (16)                        | 9                  |
| 24                                  | $M_{10}$                              | 720   | 14168  | 39, 93 (10), 157 (36),<br>160 (45)                          | 2, 7               |
| 25                                  | $S_6$                                 | 720   | 14168  | 38, 75 (6), 77 (6), 95<br>(10), 113 (15), 114<br>(15)       | 4, 5               |
| 26                                  | $L_2(11)$                             | 660   | 15456  | 104 (11), 105 (11),<br>111 (12), 175 (55)                   | 2, 7               |
| 27                                  | $2^4 : S_3 : S_3$                     | 576   | 17710  | 46, 48, 45, 51 (3), 54<br>(3), 128 (16)                     | 5, 8               |
| 28                                  | $A_4 \times 2 \times 2 : S_3 \cdot 2$ | 576   | 17710  | 47, 97 (9), 129 (16)  | 9                  |
| 29                                  | $2^4 : 3 : 2 : 2 : 3$                 | 576   | 8855   | 58, 59, 60, 55, 72 (4),<br>73 (4)                           | 10, 19             |
| 30                                  | $2^4 : 5 : 3 : 2$                     | 480   | 10626  | 50, 70, 82 (5), 143<br>(16)                                 | 10, 22             |
| 31                                  | $2^3 \cdot 2 : 2 : 2 : 3 : S_3$       | 384   | 26565  | 55, 74 (3), 83 (4), 84<br>(4), 79 (4)                       | 9, 15, 16, 19      |

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| Nr.                                 | Structure                      | Order | Length | Maximal Subgroups                              | Minimal Overgroups |
| 32                                  | $2^{1+4} : 2 : S_3$            | 384   | 26565  | 57, 53, 61, 74 (3), 81 (4)                     | 9, 11, 12          |
| 33                                  | $4^2 : 2 : 2 : S_3$            | 384   | 26565  | 56, 54, 58, 74 (3), 81 (4)                     | 11, 19             |
| 34                                  | $2 \times 4 : 2 : 2 : 2 : S_3$ | 384   | 26565  | 59, 74 (3), 85 (4), 81 (4), 86 (4)             | 12, 19             |
| 35                                  | $2^3 \cdot 2 : 2 : 2 : S_3$    | 384   | 26565  | 52, 60, 62, 74 (3), 85 (4)                     | 19                 |
| 36                                  | $A_5 : S_3$                    | 360   | 28336  | 63, 77 (3), 94 (5), 108 (6), 128 (10)          | 5, 8               |
| 37                                  | $A_6$                          | 360   | 28336  | 105 (6), 106 (6), 129 (10), 144 (15), 145 (15) | 6, 9, 13           |
| 38                                  | $A_6$                          | 360   | 14168  | 104 (6), 107 (6), 130 (10), 146 (15), 145 (15) | 6, 14 (2), 25      |
| 39                                  | $A_6$                          | 360   | 14168  | 105 (12), 129 (10), 147 (30)                   | 9 (2), 24          |
| 40                                  | $L_2(7) \times 2$              | 336   | 30360  | 65, 113 (7), 114 (7), 127 (8)                  | 4, 12              |
| 41                                  | $2^3 : 7 : 3 \times 2$         | 336   | 30360  | 68, 78, 115 (7), 127 (8)                       | 11, 12             |
| 42                                  | $2 \cdot L_2(7)$               | 336   | 30360  | 116 (7), 117 (7), 127 (8)                      | 12                 |
| 43                                  | $2^4 : 5 : 2 \cdot 2$          | 320   | 31878  | 70, 98 (5), 157 (16)                           | 15, 22             |
| 44                                  | $2^4 : 5 : 2 \cdot 2$          | 320   | 31878  | 69, 97 (5), 157 (16)                           | 16                 |
| 45                                  | $2^4 : 3 : S_3$                | 288   | 17710  | 72, 80, 82 (3), 158 (16)                       | 10, 27             |
| 46                                  | $2^4 : 3 : S_3$                | 288   | 17710  | 72, 83 (3), 85 (6), 87 (3), 94 (8)             | 19 (2), 27         |
| 47                                  | $A_4 \times 2 \times 2 : S_3$  | 288   | 17710  | 73, 79 (6), 81 (6), 94 (8)                     | 19 (2), 28         |
| 48                                  | $2^4 : 3 : S_3$                | 288   | 17710  | 72, 87, 88 (3), 158 (16)                       | 27                 |
| 49                                  | $23 : 11$                      | 253   | 40320  | 155, 182 (23)                                  | 1                  |
| 50                                  | $2^4 : 5 : 3$                  | 240   | 10626  | 91, 118, 173 (16)                              | 30                 |
| 51                                  | $2 \times 4 : 2 : 2 : S_3$     | 192   | 53130  | 88, 83, 80, 99 (3), 114 (4)                    | 15, 17, 27         |
| 52                                  | $2^3 \cdot 2 : 2 : S_3$        | 192   | 26565  | 79 (2), 89, 100 (3), 113 (4)                   | 16 (2), 35         |
| 53                                  | $2^{1+4} : S_3$                | 192   | 26565  | 90, 99 (3), 114 (8), 119 (4)                   | 17 (2), 18, 32     |

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| Nr.                                 | Structure                     | Order | Length | Maximal Subgroups               | Minimal Overgroups     |
| 54                                  | $D_8 \times 2 : 2 : S_3$      | 192   | 26565  | 82, 87 (2), 99 (3), 119 (4)     | 18, 27 (2), 33         |
| 55                                  | $4^2 : 2 : 2 : 3$             | 192   | 8855   | 101, 120 (12), 121 (4), 122 (4) | 20 (3), 21, 29, 31 (3) |
| 56                                  | $D_8 \times 2 : 2 \cdot S_3$  | 192   | 26565  | 82, 98 (3), 123 (4)             | 22 (2), 33             |
| 57                                  | $2^{1+4} : 6$                 | 192   | 26565  | 90, 100, 115 (4)                | 23 (2), 32             |
| 58                                  | $2^4 : 3 : 2 : 2$             | 192   | 8855   | 82 (3), 101, 115 (4)            | 29, 33 (3)             |
| 59                                  | $2 \times 4 : 2 : 2 : 2 : 3$  | 192   | 8855   | 101, 124 (12), 125 (4), 115 (4) | 29, 34 (3)             |
| 60                                  | $2^4 : 3 : 2 : 2$             | 192   | 8855   | 89 (3), 101, 125 (4)            | 29, 35 (3)             |
| 61                                  | $2^{1+4} \cdot S_3$           | 192   | 26565  | 90, 102 (3), 123 (4), 117 (8)   | 32                     |
| 62                                  | $2^3 \cdot 2 : 2 \cdot S_3$   | 192   | 26565  | 89, 97 (3), 126 (4)             | 35                     |
| 63                                  | $A_5 \times 3$                | 180   | 28336  | 107, 131 (5), 143 (6), 158 (10) | 10, 36                 |
| 64                                  | $L_2(7)$                      | 168   | 60720  | 145 (7), 144 (7), 156 (8)       | 6, 14, 17              |
| 65                                  | $L_2(7)$                      | 168   | 30360  | 146 (7), 145 (7), 156 (8)       | 6, 13 (2), 40          |
| 66                                  | $L_2(7)$                      | 168   | 60720  | 148 (7), 145 (7), 156 (8)       | 13, 14, 17, 18         |
| 67                                  | $2^3 : 7 : 3$                 | 168   | 60720  | 109, 149 (7), 156 (8)           | 17                     |
| 68                                  | $2^3 : 7 : 3$                 | 168   | 30360  | 110, 150 (7), 156 (8)           | 18, 41                 |
| 69                                  | $2^4 : 5 : 2$                 | 160   | 31878  | 92, 132 (5), 183 (16)           | 20, 23 (2), 44         |
| 70                                  | $2^4 : 5 : 2$                 | 160   | 10626  | 91, 133 (5), 183 (16)           | 21, 30, 43 (3)         |
| 71                                  | $3 : S_3 \cdot 2 \cdot 2 : 2$ | 144   | 70840  | 93, 96, 95, 160 (9)             | 4, 7                   |
| 72                                  | $A_4 \times 2 \times 2 : 3$   | 144   | 17710  | 125 (2), 118, 121, 131 (8)      | 29 (2), 45, 46, 48     |
| 73                                  | $2^4 : 3 : 3$                 | 144   | 17710  | 115 (2), 122 (2), 131 (8)       | 29 (2), 47             |
| 74                                  | $2^{1+4} \cdot 2^2$           | 128   | 79695  | 99, 103, 98, 102, 101, 97, 100  | 31, 32, 33, 34, 35     |
| 75                                  | $S_5$                         | 120   | 85008  | 104, 147 (5), 157 (6), 175 (10) | 7, 16, 25              |
| 76                                  | $S_5$                         | 120   | 85008  | 106, 144 (5), 157 (6), 175 (10) | 13, 15, 16             |
| 77                                  | $S_5$                         | 120   | 85008  | 107, 146 (5), 157 (6), 175 (10) | 14, 15, 25, 36         |
| 78                                  | $2^3 : 7 \times 2$            | 112   | 30360  | 110, 161, 174 (8)               | 41                     |
| 79                                  | $2^4 : S_3$                   | 96    | 53130  | 122, 132 (3), 144 (8), 146 (4)  | 20, 31 (2), 47 (2), 52 |
| 80                                  | $2^4 : S_3$                   | 96    | 17710  | 121, 133 (3), 145 (12)          | 21, 45, 51 (3)         |
| 81                                  | $2^4 : S_3$                   | 96    | 106260 | 123, 119, 115, 132 (3), 151 (4) | 23, 32, 33, 34, 47     |

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| Nr.                                 | Structure                    | Order | Length | Maximal Subgroups                                | Minimal Overgroups         |
| 82                                  | $2^4 : 3 : 2$                | 96    | 26565  | 118, 133, 150 (4)                                | 30 (2), 45 (2), 54, 56, 58 |
| 83                                  | $2^4 : S_3$                  | 96    | 53130  | 121, 134 (3), 147 (4), 144 (8)                   | 31 (2), 46, 51             |
| 84                                  | $4^2 : S_3$                  | 96    | 106260 | 120, 135 (3), 144 (4)                            | 31                         |
| 85                                  | $2^4 : S_3$                  | 96    | 106260 | 126, 113, 125, 134 (3), 151 (4)                  | 34, 35, 46                 |
| 86                                  | $Q_8 \times 2 : S_3$         | 96    | 106260 | 124, 112, 116, 136 (3), 151 (4)                  | 34                         |
| 87                                  | $2^4 : S_3$                  | 96    | 17710  | 118, 134 (3), 148 (12)                           | 46 (3), 48, 54 (3)         |
| 88                                  | $2^4 : 3 : 2$                | 96    | 53130  | 121, 134, 149 (4)                                | 48, 51                     |
| 89                                  | $2^4 : 3 : 2$                | 96    | 26565  | 122, 132, 152 (4)                                | 52, 60, 62                 |
| 90                                  | $2^{1+4} : 3$                | 96    | 26565  | 137, 153 (8), 149 (8), 150 (4)                   | 53, 57, 61                 |
| 91                                  | $2^4 : 5$                    | 80    | 10626  | 162, 198 (16)                                    | 50, 70                     |
| 92                                  | $2^4 : 5$                    | 80    | 31878  | 161, 198 (16)                                    | 69                         |
| 93                                  | $M_9$                        | 72    | 70840  | 129 (2), 130, 185 (9)                            | 6, 24 (2), 71              |
| 94                                  | $2 \times 6 : S_3$           | 72    | 141680 | 131, 146 (3), 151 (3), 144 (3), 148 (3), 159 (4) | 13, 14, 36, 46, 47         |
| 95                                  | $S_3 \times S_3 : 2$         | 72    | 70840  | 130, 128 (2), 186 (9)                            | 25 (2), 71                 |
| 96                                  | $3 : S_3 \cdot 2 \cdot 2$    | 72    | 70840  | 130, 187 (9)                                     | 71                         |
| 97                                  | $2^3 \cdot 2 : 2 \cdot 2$    | 64    | 79695  | 138, 139, 132                                    | 28 (2), 44 (2), 62, 74     |
| 98                                  | $2 \times 4 : 2 : 2 : 2$     | 64    | 79695  | 133, 139, 140                                    | 43 (2), 56, 74             |
| 99                                  | $2^{1+4} : 2$                | 64    | 79695  | 134 (2), 141 (2), 137, 133, 138                  | 51 (2), 53, 54, 74         |
| 100                                 | $2^{1+4} : 2$                | 64    | 26565  | 137, 140 (3), 132 (3)                            | 52 (3), 57, 74 (3)         |
| 101                                 | $4^2 : 2 : 2$                | 64    | 8855   | 133 (3), 142 (9), 132 (3)                        | 55, 58, 59, 60, 74 (9)     |
| 102                                 | $2^{1+4} \cdot 2$            | 64    | 79695  | 137, 136 (2), 142, 135 (2), 139                  | 61, 74                     |
| 103                                 | $2 \times 4 : 2 : 2 \cdot 2$ | 64    | 79695  | 138, 140, 142                                    | 74                         |
| 104                                 | $A_5$                        | 60    | 85008  | 176 (5), 183 (6), 196 (10)                       | 20, 26 (2), 38, 75         |
| 105                                 | $A_5$                        | 60    | 170016 | 176 (5), 183 (6), 196 (10)                       | 20, 23, 26, 37, 39         |
| 106                                 | $A_5$                        | 60    | 85008  | 177 (5), 183 (6), 196 (10)                       | 20, 21, 37 (2), 76         |
| 107                                 | $A_5$                        | 60    | 28336  | 178 (5), 183 (6), 196 (10)                       | 21, 38 (3), 63, 77 (3)     |
| 108                                 | $D_{10} \cdot S_3$           | 60    | 170016 | 143, 157 (3), 179 (5)                            | 22, 36                     |
| 109                                 | $2^3 : 7$                    | 56    | 60720  | 188, 195 (8)                                     | 67                         |
| 110                                 | $2^3 : 7$                    | 56    | 30360  | 189, 195 (8)                                     | 68, 78                     |

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| Nr.                                 | Structure               | Order | Length | Maximal Subgroups               | Minimal Overgroups                                |
| 111                                 | $11 : 5$                | 55    | 185472 | 182, 198 (11)                   | 26  |
| 112                                 | $Q_8 : S_3$             | 48    | 106260 | 154, 160 (3), 175 (4)           | 7 (2), 86   |
| 113                                 | $2^3 : S_3$             | 48    | 106260 | 146 (2), 152, 163 (3), 175 (4)  | 25 (2), 40 (2), 52, 85                            |
| 114                                 | $2^3 : S_3$             | 48    | 212520 | 145, 149, 147, 163 (3), 175 (4) | 25, 40, 51, 53                                    |
| 115                                 | $A_4 \times 2 \times 2$ | 48    | 35420  | 150 (3), 161, 180 (4)           | 41 (6), 57 (3), 58, 59, 73, 81 (3)                |
| 116                                 | $Q_8 \cdot S_3$         | 48    | 106260 | 154, 164 (3), 179 (4)           | 42 (2), 86  |
| 117                                 | $Q_8 \cdot S_3$         | 48    | 212520 | 153, 164 (3), 179 (4)           | 42, 61  |
| 118                                 | $2^4 : 3$               | 48    | 1771   | 162, 181 (20)                   | 50 (6), 72 (10), 82 (15), 87 (10)                 |
| 119                                 | $2^3 : S_3$             | 48    | 106260 | 150, 148 (2), 165 (3), 175 (4)  | 53, 54, 81  |
| 120                                 | $4^2 : 3$               | 48    | 106260 | 166, 177 (4)                    | 55, 84  |
| 121                                 | $2^4 : 3$               | 48    | 17710  | 162, 176 (12), 177 (8)          | 55 (2), 72, 80, 83 (3), 88 (3)                    |
| 122                                 | $2^4 : 3$               | 48    | 8855   | 161, 177 (16), 178 (4)          | 55 (4), 73 (4), 79 (6), 89 (3)                    |
| 123                                 | $2^3 \cdot S_3$         | 48    | 106260 | 150, 167 (3), 179 (4)           | 56, 61, 81  |
| 124                                 | $Q_8 : 6$               | 48    | 106260 | 154, 168, 180 (4)               | 59, 86  |
| 125                                 | $A_4 \times 2 \times 2$ | 48    | 35420  | 152 (3), 162, 180 (4)           | 59, 60, 72, 85 (3)                                |
| 126                                 | $2^3 \cdot S_3$         | 48    | 106260 | 152, 169 (3), 179 (4)           | 62, 85  |
| 127                                 | $7 : 3 \times 2$        | 42    | 242880 | 156, 174, 197 (7)               | 40, 41, 42  |
| 128                                 | $S_3 \times S_3$        | 36    | 141680 | 158 (2), 159, 175 (6)           | 27 (2), 36 (2), 95                                |
| 129                                 | $3 : S_3 \cdot 2$       | 36    | 141680 | 159, 199 (9)                    | 28 (2), 37 (2), 39, 93                            |
| 130                                 | $3 : S_3 \cdot 2$       | 36    | 70840  | 159, 199 (9)                    | 38 (2), 93, 95, 96                                |
| 131                                 | $A_4 \times 3$          | 36    | 141680 | 178, 180, 177, 181, 184 (4)     | 63, 72, 73, 94                                    |
| 132                                 | $2^3 \cdot 2 : 2$       | 32    | 26565  | 165 (3), 167 (3), 161           | 69 (6), 79 (6), 81 (12), 89, 97 (3), 100 (3), 101 |
| 133                                 | $2 \times 4 : 2 : 2$    | 32    | 26565  | 162, 165 (3), 170 (3)           | 70 (2), 80 (2), 82, 98 (3), 99 (3), 101           |
| 134                                 | $D_8 \times 2 : 2$      | 32    | 53130  | 162, 169 (3), 163 (3)           | 83 (3), 85 (6), 87, 88, 99 (3)                    |
| 135                                 | $D_8 : 4$               | 32    | 159390 | 171, 172, 166                   | 84 (2), 102                                       |
| 136                                 | $Q_8 \cdot 2^2$         | 32    | 159390 | 160 (2), 172, 171, 164 (2), 168 | 86 (2), 102                                       |
| 137                                 | $2^{1+4}$               | 32    | 26565  | 163 (6), 171 (6), 165 (3)       | 90, 99 (3), 100, 102 (3)                          |
| 138                                 | $D_8 \times 2 \cdot 2$  | 32    | 79695  | 169 (2), 165                    | 97, 99, 103                                       |
| 139                                 | $8 : 2 : 2$             | 32    | 79695  | 165, 172 (2)                    | 97, 98, 102                                       |

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| Nr.                                 | Structure            | Order | Length | Maximal Subgroups               | Minimal Overgroups                                  |
| 140                                 | $2 \times 4 : 2 : 2$ | 32    | 79695  | 165, 167 (2)                    | 98, 100, 103  |
| 141                                 | $2^3 \cdot 2 : 2$    | 32    | 159390 | 169, 163, 170                   | 99  |
| 142                                 | $2 \times 4 : 2 : 2$ | 32    | 79695  | 170 (2), 167 (2), 165, 166, 168 | 101, 102, 103                                       |
| 143                                 | $15 : 2$             | 30    | 170016 | 173, 183, 197 (5)               | 30, 63, 108   |
| 144                                 | $2^2 : S_3$          | 24    | 425040 | 177, 190 (3), 196 (4)           | 37, 64, 76, 79, 83, 84, 94                          |
| 145                                 | $2^2 : S_3$          | 24    | 212520 | 176, 190 (3), 196 (4)           | 37 (2), 38, 64 (2), 65, 66 (2), 80, 114             |
| 146                                 | $2^2 : S_3$          | 24    | 212520 | 178, 190 (3), 196 (4)           | 38, 65, 77 (2), 79, 94 (2), 113                     |
| 147                                 | $2^2 : S_3$          | 24    | 212520 | 176, 186 (3), 196 (4)           | 39 (2), 75 (2), 83, 114                             |
| 148                                 | $2^2 : S_3$          | 24    | 212520 | 181, 190 (3), 196 (4)           | 66 (2), 87, 94 (2), 119                             |
| 149                                 | $A_4 \times 2$       | 24    | 212520 | 176, 188, 197 (4)               | 67 (2), 88, 90, 114                                 |
| 150                                 | $A_4 \times 2$       | 24    | 106260 | 181, 189, 197 (4)               | 68 (2), 82, 90, 115, 119, 123                       |
| 151                                 | $2^2 : S_3$          | 24    | 425040 | 180, 179, 175, 190 (3)          | 81, 85, 86, 94                                      |
| 152                                 | $A_4 \times 2$       | 24    | 106260 | 178, 191, 197 (4)               | 89, 113, 125, 126                                   |
| 153                                 | $Q_8 : 3$            | 24    | 212520 | 192, 197 (4)                    | 90, 117   |
| 154                                 | $Q_8 : 3$            | 24    | 106260 | 185, 197 (4)                    | 112, 116, 124                                       |
| 155                                 | $23$                 | 23    | 40320  | 204                             | 49  |
| 156                                 | $7 : 3$              | 21    | 242880 | 195, 202 (7)                    | 64 (2), 65, 66 (2), 67 (2), 68, 127                 |
| 157                                 | $D_{10} \cdot 2$     | 20    | 510048 | 183, 199 (5)                    | 24, 43, 44, 75, 76, 77, 108                         |
| 158                                 | $S_3 \times 3$       | 18    | 283360 | 184, 196, 197 (3)               | 45, 48, 63, 128                                     |
| 159                                 | $3 : S_3$            | 18    | 70840  | 184, 196 (12)                   | 94 (8), 128 (2), 129 (2), 130                       |
| 160                                 | $Q_8 : 2$            | 16    | 318780 | 186, 187, 185                   | 24 (2), 71 (2), 112, 136                            |
| 161                                 | $2^4$                | 16    | 253    | 189 (15)                        | 78 (120), 92 (126), 115 (140), 122 (35), 132 (105)  |
| 162                                 | $2^4$                | 16    | 1771   | 191 (15)                        | 91 (6), 118, 121 (10), 125 (20), 133 (15), 134 (30) |
| 163                                 | $D_8 \times 2$       | 16    | 159390 | 191, 190 (2), 186 (2), 193, 188 | 113 (2), 114 (4), 134, 137, 141                     |
| 164                                 | $Q_{16}$             | 16    | 318780 | 192, 187, 185                   | 116, 117 (2), 136                                   |
| 165                                 | $D_8 \times 2$       | 16    | 79695  | 191, 190 (4), 194, 189          | 119 (4), 132, 133, 137, 138, 139, 140, 142          |

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|-------------------------------------|------------------|-------|--------|---------------------------------|--|
| Nr.                                 | Structure        | Order | Length | Maximal Subgroups               | Minimal Overgroups   |
| 166                                 | $4^2$            | 16    | 26565  | 194 (3)                         | 120 (4), 135 (6), 142 (3)  |
| 167                                 | $2 \times 4 : 2$ | 16    | 79695  | 189, 194 (2)                    | 123 (4), 132, 140 (2), 142 (2)                                       |
| 168                                 | $Q_8 \times 2$   | 16    | 26565  | 194 (3), 185 (4)                | 124 (4), 136 (6), 142 (3)  |
| 169                                 | $2 \times 4 : 2$ | 16    | 159390 | 191, 193 (2)                    | 126 (2), 134, 138, 141   |
| 170                                 | $2 \times 4 : 2$ | 16    | 79695  | 191, 194 (2)                    | 133, 141 (2), 142 (2)  |
| 171                                 | $D_8 : 2$        | 16    | 159390 | 190 (2), 193 (2), 186, 192, 194 | 135, 136, 137  |
| 172                                 | $8 : 2$          | 16    | 159390 | 194, 187 (2)                    | 135, 136, 139  |
| 173                                 | 15               | 15    | 170016 | 198, 202                        | 50, 143  |
| 174                                 | 14               | 14    | 242880 | 195, 203                        | 78, 127  |
| 175                                 | $D_{12}$         | 12    | 425040 | 196 (2), 197, 200 (3)           | 26 (2), 75 (2), 76 (2), 77 (2), 112, 113, 114 (2), 119, 128 (2), 151 |
| 176                                 | $A_4$            | 12    | 212520 | 200, 202 (4)                    | 104 (2), 105 (4), 121, 145, 147, 149                                 |
| 177                                 | $A_4$            | 12    | 141680 | 201, 202 (4)                    | 106 (3), 120 (3), 121, 122, 131, 144 (3)                             |
| 178                                 | $A_4$            | 12    | 35420  | 201, 202 (4)                    | 107 (4), 122, 131 (4), 146 (6), 152 (3)                              |
| 179                                 | $Q_{12}$         | 12    | 425040 | 197, 199 (3)                    | 108 (2), 116, 117 (2), 123, 126, 151                                 |
| 180                                 | $2 \times 6$     | 12    | 141680 | 197 (3), 201                    | 115, 124 (3), 125, 131, 151 (3)                                      |
| 181                                 | $A_4$            | 12    | 35420  | 201, 202 (4)                    | 118, 131 (4), 148 (6), 150 (3)                                       |
| 182                                 | 11               | 11    | 185472 | 204                             | 49 (5), 111  |
| 183                                 | $D_{10}$         | 10    | 170016 | 198, 203 (5)                    | 69 (3), 70, 104 (3), 105 (6), 106 (3), 107, 143, 157 (3)             |
| 184                                 | $3^2$            | 9     | 70840  | 202 (4)                         | 131 (8), 158 (4), 159  |
| 185                                 | $Q_8$            | 8     | 106260 | 199 (3)                         | 93 (6), 154, 160 (3), 164 (3), 168                                   |
| 186                                 | $D_8$            | 8     | 159390 | 200 (2), 199                    | 95 (4), 147 (4), 160 (2), 163 (2), 171                               |
| 187                                 | 8                | 8     | 318780 | 199                             | 96 (2), 160, 164, 172  |
| 188                                 | $2^3$            | 8     | 7590   | 200 (7)                         | 109 (8), 149 (28), 163 (21)  |
| 189                                 | $2^3$            | 8     | 3795   | 201 (7)                         | 110 (8), 150 (28), 161, 165 (21), 167 (21)                           |

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|-------------------------------------|--------------|-------|--------|-------------------|--|
| Nr.                                 | Structure    | Order | Length | Maximal Subgroups | Minimal Overgroups   |
| 190                                 | $D_8$        | 8     | 318780 | 200, 199, 201     | 144 (4), 145 (2), 146 (2), 148 (2), 151 (4), 163, 165, 171   |
| 191                                 | $2^3$        | 8     | 26565  | 200 (6), 201      | 152 (4), 162, 163 (6), 165 (3), 169 (6), 170 (3)   |
| 192                                 | $Q_8$        | 8     | 53130  | 199 (3)           | 153 (4), 164 (6), 171 (3)  |
| 193                                 | $2 \times 4$ | 8     | 159390 | 199 (2), 200      | 163, 169 (2), 171 (2)  |
| 194                                 | $2 \times 4$ | 8     | 79695  | 199 (2), 201      | 165, 166, 167 (2), 168, 170 (2), 171 (2), 172 (2)  |
| 195                                 | 7            | 7     | 242880 | 204               | 109 (2), 110, 156, 174   |
| 196                                 | $S_3$        | 6     | 283360 | 202, 203 (3)      | 104 (3), 105 (6), 106 (3), 107, 144 (6), 145 (3), 146 (3), 147 (3), 148 (3), 158, 159 (3), 175 (3) |
| 197                                 | 6            | 6     | 425040 | 202, 203          | 127 (4), 143 (2), 149 (2), 150, 152, 153 (2), 154, 158 (2), 175, 179, 180                          |
| 198                                 | 5            | 5     | 170016 | 204               | 91, 92 (3), 111 (12), 173, 183   |
| 199                                 | 4            | 4     | 159390 | 203               | 129 (8), 130 (4), 157 (16), 179 (8), 185 (2), 186, 187 (2), 190 (2), 192, 193 (2), 194             |
| 200                                 | $2^2$        | 4     | 53130  | 203 (3)           | 175 (24), 176 (4), 186 (6), 188, 190 (6), 191 (3), 193 (3)   |
| 201                                 | $2^2$        | 4     | 8855   | 203 (3)           | 177 (16), 178 (4), 180 (16), 181 (4), 189 (3), 190 (36), 191 (3), 194 (9)                          |
| 202                                 | 3            | 3     | 28336  | 204               | 156 (60), 173 (6), 176 (30), 177 (20), 178 (5), 181 (5), 184 (10), 196 (10), 197 (15)              |
| 203                                 | 2            | 2     | 3795   | 204               | 174 (64), 183 (224), 196 (224), 197 (112), 199 (42), 200 (42), 201 (7)                             |

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|-------------------------------------|-----------|-------|--------|-------------------|--|
| Nr.                                 | Structure | Order | Length | Maximal Subgroups | Minimal Overgroups   |
| 204                                 | 1         | 1     | 1      |                   | 155 (40320), 182<br>(185472), 195<br>(242880), 198<br>(170016), 202<br>(28336), 203 (3795) |

TABLE 1. Subgroup lattice of  $M_{23}$ 

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