

THE SUBGROUP LATTICE OF M_{22}

THOMAS CONNOR AND DIMITRI LEEMANS

Nr.	Structure	Order	Length	Maximal Subgroups	Minimal Overgroups
1	M_{22}	443520	1	2 (22), 3 (77), 4 (176), 5 (176), 6 (231), 7 (330), 11 (616), 12 (672)	
2	$L_3(4)$	20160	22	8 (21), 9 (21), 16 (56), 17 (56), 18 (56), 27 (120), 28 (120), 29 (120), 47 (280)	1
3	$2^4 : A_6$	5760	77	10 (6), 8 (6), 13 (10), 14 (15), 15 (15), 16 (16), 19 (16)	1
4	A_7	2520	176	18 (7), 30 (15), 27 (15), 36 (21), 48 (35)	1
5	A_7	2520	176	17 (7), 29 (15), 30 (15), 37 (21), 48 (35)	1
6	$2^4 : A_5 : 2$	1920	231	9, 14 (5), 20 (6), 22 (10), 37 (16), 36 (16)	1
7	$2^3 : L_2(7)$	1344	330	23 (7), 22 (7), 30 (8), 28 (8), 31 (8)	1
8	$2^4 : A_5$	960	462	24 (5), 32 (6), 38 (10), 56 (16), 57 (16), 58 (16), 59 (16)	2, 3
9	$2^4 : A_5$	960	231	24 (5), 33 (6), 39 (10), 60 (16), 56 (32), 61 (16)	2 (2), 6
10	$2^4 : A_5$	960	462	25 (5), 32 (6), 40 (10), 59 (16)	3
11	M_{10}	720	616	19, 47 (10), 101 (36), 103 (45)	1
12	$L_2(11)$	660	672	57 (11), 58 (11), 63 (12), 120 (55)	1
13	$2^4 : 3 : S_3 \cdot 2$	576	770	21, 49 (9), 73 (16)	3
14	$2 \times 4 : 2 : 2 : 2 : S_3$	384	1155	24, 35 (3), 41 (4), 38 (4), 42 (4)	3, 6

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Nr.	Structure	Order	Length	Maximal Subgroups	Minimal Overgroups
15	$2^{1+4} : 2 : S_3$	384	1155	26, 23, 25, 35 (3), 40 (4)	3
16	A_6	360	1232	59 (6), 56 (6), 73 (10), 89 (15), 90 (15)	2, 3
17	A_6	360	1232	58 (6), 61 (6), 74 (10), 91 (15), 92 (15)	2, 5
18	A_6	360	1232	57 (6), 60 (6), 75 (10), 93 (15), 92 (15)	2, 4
19	A_6	360	616	59 (12), 73 (10), 94 (30)	3 (2), 11
20	$2^4 : 5 : 2 \cdot 2$	320	1386	33, 50 (5), 101 (16)	6
21	$A_4 \times 2 \times 2 : S_3$	288	770	34, 38 (6), 40 (6), 48 (8)	13
22	$2 \times 4 : 2 : 2 : S_3$	192	2310	43, 41, 39, 51 (3), 64 (4)	6, 7
23	$2^{1+4} : S_3$	192	1155	44, 51 (3), 64 (8), 65 (4)	7 (2), 15
24	$Q_8 \times 2 : 2 : 2 : 3$	192	1155	52, 66 (8), 67 (4), 68 (4), 69 (4)	8 (2), 9, 14
25	$2^{1+4} : 6$	192	1155	44, 53, 70 (4)	10 (2), 15
26	$2^{1+4} \cdot S_3$	192	1155	44, 54 (3), 71 (8), 72 (4)	15
27	$L_2(7)$	168	2640	91 (7), 92 (7), 100 (8)	2, 4
28	$L_2(7)$	168	2640	89 (7), 90 (7), 100 (8)	2, 7
29	$L_2(7)$	168	2640	92 (7), 93 (7), 100 (8)	2, 5
30	$L_2(7)$	168	2640	95 (7), 92 (7), 100 (8)	4, 5, 7
31	$2^3 : 7 : 3$	168	2640	62, 96 (7), 100 (8)	7
32	$2^4 : 5 : 2$	160	2772	45, 77 (5), 130 (16)	8, 10
33	$2^4 : 5 : 2$	160	1386	46, 78 (5), 130 (16)	9, 20
34	$A_4 \times 2 \times 2 : 3$	144	770	69 (2), 70 (2), 76 (8)	21
35	$2^{1+4} \cdot 2^2$	128	3465	51, 49, 50, 53, 52, 55, 54	14, 15
36	S_5	120	3696	60, 93 (5), 101 (6), 120 (10)	4, 6
37	S_5	120	3696	61, 91 (5), 101 (6), 120 (10)	5, 6
38	$2^4 : S_3$	96	4620	69, 77 (3), 89 (4), 91 (4), 93 (4)	8, 14, 21
39	$2^4 : S_3$	96	2310	68, 78 (3), 92 (8), 90 (4)	9, 22
40	$2^4 : S_3$	96	4620	70, 65, 72, 77 (3), 97 (4)	10, 15, 21

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Nr.	Structure	Order	Length	Maximal Subgroups	Minimal Overgroups
41	$2^4 : S_3$	96	2310	68, 79 (3), 89 (8), 94 (4)	14 (2), 22
42	$4^2 : S_3$	96	4620	67, 80 (3), 89 (4)	14
43	$2^4 : 3 : 2$	96	2310	68, 79, 96 (4)	22
44	$2^{1+4} : 3$	96	1155	81, 98 (4), 96 (8), 99 (8)	23, 25, 26
45	$2^4 : 5$	80	2772	104, 147 (16)	32
46	$2^4 : 5$	80	1386	105, 147 (16)	33
47	M_9	72	6160	74, 73, 75, 132 (9)	2, 11
48	$A_4 : S_3$	72	6160	76, 97 (3), 91 (3), 93 (3), 95 (3), 102 (4)	4, 5, 21
49	$2^3 \cdot 2 : 2 : 2$	64	3465	82, 83, 77	13 (2), 35
50	$2 \times 4 : 2 : 2 : 2$	64	3465	78, 83, 84	20 (2), 35
51	$2^{1+4} : 2$	64	3465	79 (2), 78, 85 (2), 82, 81	22 (2), 23, 35
52	$Q_8 \times 2 : 2 : 2$	64	1155	78 (3), 86 (3), 87 (6), 77 (3)	24, 35 (3)
53	$2^{1+4} : 2$	64	1155	81, 84 (3), 77 (3)	25, 35 (3)
54	$2^{1+4} \cdot 2$	64	3465	81, 83, 88 (2), 80 (2), 86	26, 35
55	$2 \times 4 : 2 : 2 \cdot 2$	64	3465	82, 84, 86	35
56	A_5	60	7392	121 (5), 130 (6), 145 (10)	8, 9, 16
57	A_5	60	7392	122 (5), 130 (6), 145 (10)	8, 12, 18
58	A_5	60	7392	122 (5), 130 (6), 145 (10)	8, 12, 17
59	A_5	60	7392	123 (5), 130 (6), 145 (10)	8, 10, 16, 19
60	A_5	60	3696	124 (5), 130 (6), 145 (10)	9, 18 (2), 36
61	A_5	60	3696	125 (5), 130 (6), 145 (10)	9, 17 (2), 37
62	$2^3 : 7$	56	2640	133, 144 (8)	31
63	$11 : 5$	55	8064	129, 147 (11)	12
64	$2^3 : S_3$	48	9240	94, 96, 90, 106 (3), 120 (4)	22, 23
65	$2^3 : S_3$	48	4620	98, 95 (2), 107 (3), 120 (4)	23, 40
66	$4^2 : 3$	48	9240	108, 121 (4)	24
67	$4^2 : 3$	48	4620	109, 121 (4)	24, 42
68	$2^4 : 3$	48	2310	105, 123 (4), 122 (8), 121 (8)	24 (2), 39, 41, 43

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Nr.	Structure	Order	Length	Maximal Subgroups	Minimal Overgroups
69	$2^4 : 3$	48	1540	104, 121 (12), 124 (4), 125 (4)	24 (3), 34, 38 (3)
70	$A_4 \times 2 \times 2$	48	1540	98 (3), 104, 126 (4)	25 (3), 34, 40 (3)
71	$Q_8 \cdot S_3$	48	9240	99, 110 (3), 127 (4)	26
72	$2^3 \cdot S_3$	48	4620	98, 111 (3), 127 (4)	26, 40
73	$3 : S_3 \cdot 2$	36	6160	102, 148 (9)	13 (2), 16 (2), 19, 47
74	$3 : S_3 \cdot 2$	36	6160	102, 149 (9)	17 (2), 47
75	$3 : S_3 \cdot 2$	36	6160	102, 149 (9)	18 (2), 47
76	$A_4 \times 3$	36	6160	124, 125, 126, 128, 131 (4)	34, 48
77	$2^3 \cdot 2 : 2$	32	3465	107 (2), 111 (2), 112, 113, 104	32 (4), 38 (4), 40 (4), 49, 52, 53
78	$D_8 \times 2 : 2$	32	3465	105, 114 (2), 115, 112, 107 (2)	33 (2), 39 (2), 50, 51, 52
79	$D_8 \times 2 : 2$	32	2310	105, 116 (3), 106 (3)	41 (3), 43, 51 (3)
80	$D_8 : 4$	32	6930	117, 118, 109	42 (2), 54
81	2^{1+4}	32	1155	106 (6), 117 (6), 112 (3)	44, 51 (3), 53, 54 (3)
82	$D_8 \times 2 \cdot 2$	32	3465	116 (2), 112	49, 51, 55
83	$8 : 2 : 2$	32	3465	112, 118 (2)	49, 50, 54
84	$2 \times 4 : 2 : 2$	32	3465	112, 113 (2)	50, 53, 55
85	$2 \times 4 : 2 : 2$	32	6930	106, 116, 115	51
86	$2 \times 4 : 2 : 2$	32	3465	114 (2), 111 (2), 112, 119, 109	52, 54, 55
87	$2 \times 4 : 2 : 2$	32	6930	115, 108, 107, 113, 114, 111, 119	52
88	$Q_8 \cdot 2^2$	32	6930	103 (2), 110 (2), 117, 119, 118	54
89	$2^2 : S_3$	24	18480	121, 134 (3), 145 (4)	16, 28, 38, 41, 42
90	$2^2 : S_3$	24	9240	123, 134 (3), 145 (4)	16 (2), 28 (2), 39, 64
91	$2^2 : S_3$	24	18480	125, 135 (3), 145 (4)	17, 27, 37, 38, 48
92	$2^2 : S_3$	24	18480	122, 135 (3), 145 (4)	17, 18, 27, 29, 30, 39
93	$2^2 : S_3$	24	18480	124, 135 (3), 145 (4)	18, 29, 36, 38, 48
94	$2^2 : S_3$	24	9240	123, 136 (3), 145 (4)	19 (2), 41, 64
95	$2^2 : S_3$	24	9240	128, 135 (3), 145 (4)	30 (2), 48 (2), 65
96	$A_4 \times 2$	24	9240	123, 133, 146 (4)	31 (2), 43, 44, 64
97	$2^2 : S_3$	24	18480	127, 126, 120, 135 (3)	40, 48
98	$A_4 \times 2$	24	4620	128, 137, 146 (4)	44, 65, 70, 72
99	$Q_8 : 3$	24	9240	138, 146 (4)	44, 71
100	$7 : 3$	21	21120	144, 154 (7)	27, 28, 29, 30, 31
101	$D_{10} \cdot 2$	20	22176	130, 149 (5)	11, 20, 36, 37
102	$3 : S_3$	18	6160	131, 145 (12)	48 (4), 73, 74, 75
103	$Q_8 : 2$	16	13860	136, 139, 132	11 (2), 88

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Nr.	Structure	Order	Length	Maximal Subgroups	Minimal Overgroups
104	2^4	16	77	137 (15)	45 (36), 69 (20), 70 (20), 77 (45)
105	2^4	16	231	140 (15)	46 (6), 68 (10), 78 (15), 79 (10)
106	$D_8 \times 2$	16	6930	136 (2), 134 (2), 140, 133, 141	64 (4), 79, 81, 85
107	$D_8 \times 2$	16	6930	140, 135 (4), 142, 137	65 (2), 77, 78, 87
108	4^2	16	2310	142 (3)	66 (4), 87 (3)
109	4^2	16	1155	143 (3)	67 (4), 80 (6), 86 (3)
110	Q_{16}	16	13860	132, 139, 138	71 (2), 88
111	$2 \times 4 : 2$	16	6930	137, 142, 143	72 (2), 77, 86, 87
112	$D_8 \times 2$	16	3465	134 (4), 140, 143, 137	77, 78, 81, 82, 83, 84, 86
113	$2 \times 4 : 2$	16	3465	137, 142 (2)	77, 84 (2), 87 (2)
114	$2 \times 4 : 2$	16	6930	140, 142, 143	78, 86, 87
115	$2 \times 4 : 2$	16	3465	140, 142 (2)	78, 85 (2), 87 (2)
116	$2 \times 4 : 2$	16	6930	140, 141 (2)	79, 82, 85
117	$D_8 : 2$	16	6930	134 (2), 141 (2), 136, 138, 143	80, 81, 88
118	$8 : 2$	16	6930	143, 139 (2)	80, 83, 88
119	$Q_8 \times 2$	16	3465	132 (4), 142 (2), 143	86, 87 (2), 88 (2)
120	D_{12}	12	18480	146, 145 (2), 150 (3)	12 (2), 36 (2), 37 (2), 64 (2), 65, 97
121	A_4	12	18480	151, 154 (4)	56 (2), 66 (2), 67, 68, 69, 89
122	A_4	12	18480	150, 154 (4)	57 (2), 58 (2), 68, 92
123	A_4	12	9240	152, 154 (4)	59 (4), 68, 90, 94, 96
124	A_4	12	6160	153, 154 (4)	60 (3), 69, 76, 93 (3)
125	A_4	12	6160	153, 154 (4)	61 (3), 69, 76, 91 (3)
126	2×6	12	6160	146 (3), 153	70, 76, 97 (3)
127	Q_{12}	12	18480	146, 149 (3)	71 (2), 72, 97
128	A_4	12	1540	153, 154 (4)	76 (4), 95 (6), 98 (3)
129	11	11	8064	156	63
130	D_{10}	10	22176	147, 155 (5)	32 (2), 33, 56 (2), 57 (2), 58 (2), 59 (2), 60, 61, 101
131	3^2	9	6160	154 (4)	76 (4), 102
132	Q_8	8	13860	149 (2), 148	47 (4), 103, 110, 119
133	2^3	8	330	152 (7)	62 (8), 96 (28), 106 (21)
134	D_8	8	13860	152, 148, 151	89 (4), 90 (2), 106, 112, 117

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Nr.	Structure	Order	Length	Maximal Subgroups	Minimal Overgroups
135	D_8	8	27720	150, 149, 153	91 (2), 92 (2), 93 (2), 95, 97 (2), 107
136	D_8	8	6930	152 (2), 148	94 (4), 103 (2), 106 (2), 117
137	2^3	8	1155	151 (3), 153 (4)	98 (4), 104, 107 (6), 111 (6), 112 (3), 113 (3)
138	Q_8	8	2310	148 (3)	99 (4), 110 (6), 117 (3)
139	8	8	13860	148	103, 110, 118
140	2^3	8	3465	151, 150 (4), 152 (2)	105, 106 (2), 107 (2), 112, 114 (2), 115, 116 (2)
141	2×4	8	6930	148 (2), 152	106, 116 (2), 117 (2)
142	2×4	8	6930	149 (2), 151	107, 108, 111, 113, 114, 115, 119
143	2×4	8	3465	148 (2), 151	109, 111 (2), 112, 114 (2), 117 (2), 118 (2), 119
144	7	7	21120	156	62, 100
145	S_3	6	36960	154, 155 (3)	56 (2), 57 (2), 58 (2), 59 (2), 60, 61, 89 (2), 90, 91 (2), 92 (2), 93 (2), 94, 95, 102 (2), 120
146	6	6	18480	154, 155	96 (2), 98, 99 (2), 120, 126, 127
147	5	5	22176	156	45 (2), 46, 63 (4), 130
148	4	4	6930	155	73 (8), 132 (2), 134 (2), 136, 138, 139 (2), 141 (2), 143
149	4	4	13860	155	74 (4), 75 (4), 101 (8), 127 (4), 132 (2), 135 (2), 142
150	2^2	4	4620	155 (3)	120 (12), 122 (4), 135 (6), 140 (3)
151	2^2	4	1155	155 (3)	121 (16), 134 (12), 137 (3), 140 (3), 142 (6), 143 (3)
152	2^2	4	2310	155 (3)	123 (4), 133, 134 (6), 136 (6), 140 (3), 141 (3)

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Nr.	Structure	Order	Length	Maximal Subgroups	Minimal Overgroups
153	2^2	4	1540	155 (3)	124 (4), 125 (4), 126 (4), 128, 135 (18), 137 (3)
154	3	3	6160	156	100 (24), 121 (12), 122 (12), 123 (6), 124 (4), 125 (4), 128, 131 (4), 145 (6), 146 (3)
155	2	2	1155	156	130 (96), 145 (96), 146 (16), 148 (6), 149 (12), 150 (12), 151 (3), 152 (6), 153 (4)
156	1	1	1		129 (8064), 144 (21120), 147 (22176), 154 (6160), 155 (1155)

TABLE 1. Subgroup lattice of M_{22}

UNIVERSITÉ LIBRE DE BRUXELLES, DÉPARTEMENT DE MATHÉMATIQUES - C.P.216, BOULEVARD DU TRIOMPHE,
B-1050 BRUXELLES, BOURSIER FR1A
E-mail address: tconnor@ulb.ac.be

UNIVERSITY OF AUCKLAND, DEPARTMENT OF MATHEMATICS, PRIVATE BAG 92019, AUCKLAND, NEW ZEALAND
E-mail address: d.leemans@auckland.ac.nz