

THE SUBGROUP LATTICE OF M_{12}

THOMAS CONNOR AND DIMITRI LEEMANS

Nr.	Structure	Order	Length	Maximal Subgroups	Minimal Overgroups
1	M_{12}	95040	1	2 (12), 3 (12), 4 (66), 5 (66), 12 (144), 14 (220), 15 (220), 18 (396), 21 (495), 22 (495), 35 (1320)	
2	M_{11}	7920	12	6 (11), 13 (12), 23 (55), 25 (66), 51 (165)	1
3	M_{11}	7920	12	7 (11), 13 (12), 24 (55), 26 (66), 52 (165)	1
4	$A_6 : 2 : 2$	1440	66	7, 8, 9, 24 (10), 56 (36), 62 (45)	1
5	$S_6 : 2$	1440	66	6, 10, 11, 23 (10), 56 (36), 63 (45)	1
6	M_{10}	720	66	16, 36 (10), 80 (36), 88 (45)	2 (2), 5
7	M_{10}	720	66	17, 37 (10), 81 (36), 89 (45)	3 (2), 4
8	$A_6 : 2$	720	66	17, 38 (10), 82 (36), 90 (45)	4
9	$A_6 : 2$	720	66	17, 25 (12), 39 (10), 53 (30)	4
10	$A_6 : 2$	720	66	16, 40 (10), 82 (36), 91 (45)	5
11	S_6	720	66	16, 26 (12), 41 (10), 53 (30)	5
12	$L_2(11)$	660	144	43 (22), 47 (12), 100 (55)	1
13	$L_2(11)$	660	144	44 (11), 45 (11), 47 (12), 101 (55)	2, 3
14	$M_9 : S_3$	432	220	19, 24 (3), 30 (4), 52 (9)	1

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Nr.	Structure	Order	Length	Maximal Subgroups	Minimal Overgroups
15	$M_9 : S_3$	432	220	20, 23 (3), 30 (4), 51 (9)	1
16	A_6	360	66	44 (12), 57 (10), 70 (30)	6, 10, 11
17	A_6	360	66	45 (12), 58 (10), 71 (30)	7, 8, 9
18	$A_5 : 2 \times 2$	240	396	27, 28, 29, 54 (5), 56 (6), 72 (10)	1
19	$M_9 : 3$	216	220	37, 48 (4), 73 (9)	14
20	$M_9 : 3$	216	220	36, 49 (4), 74 (9)	15
21	$D_8 \times 2 : 2 : S_3$	192	495	31, 32, 33, 42 (3), 54 (4)	1
22	$2^{1+4} : S_3$	192	495	34, 42 (3), 53 (4), 52 (4), 51 (4)	1
23	$M_9 : 2$	144	660	40, 36, 41, 88 (9)	2, 5, 15
24	$M_9 : 2$	144	660	38, 37, 39, 89 (9)	3, 4, 14
25	$A_5 : 2$	120	792	45, 70 (5), 80 (6), 101 (10)	2, 9
26	S_5	120	792	44, 71 (5), 81 (6), 101 (10)	3, 11
27	$A_5 : 2$	120	396	46, 75 (5), 81 (6), 102 (10)	18
28	$A_5 : 2$	120	396	46, 76 (5), 80 (6), 102 (10)	18
29	$A_5 \times 2$	120	396	46, 77 (5), 82 (6), 102 (10)	18
30	$3^{1+2} : 2^2$	108	880	48, 50, 49, 59 (3), 60 (3)	14, 15
31	$4^2 : S_3$	96	495	55, 64 (3), 76 (4)	21
32	$4^2 : 3 : 2$	96	495	55, 65, 77 (4)	21
33	$4^2 : S_3$	96	495	55, 66 (3), 75 (4)	21
34	$2^{1+4} : 3$	96	495	67, 74 (4), 78 (4), 73 (4), 79 (8)	22
35	$A_4 \times S_3$	72	1320	61, 72, 79 (3), 83 (4)	1
36	M_9	72	220	57 (3), 115 (9)	6 (3), 20, 23 (3)
37	M_9	72	220	58 (3), 116 (9)	7 (3), 19, 24 (3)
38	$3 : S_3 \cdot 2 \cdot 2$	72	660	58, 117 (9)	8, 24
39	$S_3 \times S_3 : 2$	72	660	58, 60 (2), 118 (9)	9, 24
40	$3 : S_3 \cdot 2 \cdot 2$	72	660	57, 119 (9)	10, 23
41	$S_3 \times S_3 : 2$	72	660	57, 59 (2), 120 (9)	11, 23
42	$2^{1+4} : 2$	64	1485	65, 66, 67, 63, 68, 64, 62	21, 22

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Nr.	Structure	Order	Length	Maximal Subgroups	Minimal Overgroups
43	A_5	60	1584	103 (5), 109 (6), 130 (10)	12 (2)
44	A_5	60	792	104 (5), 110 (6), 131 (10)	13 (2), 16, 26
45	A_5	60	792	104 (5), 110 (6), 132 (10)	13 (2), 17, 25
46	A_5	60	396	105 (5), 110 (6), 133 (10)	27, 28, 29
47	$11 : 5$	55	1728	108, 136 (11)	12, 13
48	$3^{1+2} : 2$	54	880	69, 84, 85 (3)	19, 30
49	$3^{1+2} : 2$	54	880	69, 86, 87 (3)	20, 30
50	$3^{1+2} : 2$	54	880	69, 85 (3), 83 (6), 87 (3)	30
51	$Q_8 : S_3$	48	1980	74, 88 (3), 101 (4)	2, 15, 22
52	$Q_8 : S_3$	48	1980	73, 89 (3), 101 (4)	3, 14, 22
53	$2^3 : S_3$	48	1980	71, 78, 70, 92 (3), 101 (4)	9, 11, 22
54	$2^3 : S_3$	48	1980	76, 77, 75, 93 (3), 100 (4)	18, 21
55	$4^2 : 3$	48	495	94, 105 (4)	31, 32, 33
56	$D_{10} \cdot 2 \times 2$	40	2376	80, 82, 81, 121 (5)	4, 5, 18
57	$3 : S_3 \cdot 2$	36	660	86, 137 (9)	16, 36, 40, 41
58	$3 : S_3 \cdot 2$	36	660	84, 138 (9)	17, 37, 38, 39
59	$S_3 \times S_3$	36	1320	85 (2), 86, 101 (6)	30 (2), 41
60	$S_3 \times S_3$	36	1320	84, 87 (2), 101 (6)	30 (2), 39
61	$A_4 \times 3$	36	1320	103 (2), 106, 107, 112 (4)	35
62	$Q_8 : 2^2$	32	1485	92, 89 (2), 95, 96, 90 (2)	4 (2), 42
63	$Q_8 : 2^2$	32	1485	92, 88 (2), 97, 98, 91 (2)	5 (2), 42
64	$D_8 : 4$	32	1485	94, 98, 95	31, 42
65	$D_8 \times 2 : 2$	32	495	92 (3), 99 (3), 94	32, 42 (3)
66	$Q_8 : 4$	32	1485	94, 97, 96	33, 42
67	2^{1+4}	32	495	99 (3), 96 (3), 93 (6), 98 (3)	34, 42 (3)
68	$8 : 2 : 2$	32	1485	99, 97, 95	42
69	3^{1+2}	27	880	113, 112 (2), 114	48, 49, 50
70	$2^2 : S_3$	24	1980	104, 120 (3), 131 (4)	16, 25 (2), 53
71	$2^2 : S_3$	24	1980	104, 118 (3), 132 (4)	17, 26 (2), 53
72	$S_3 \times 2^2$	24	1320	102 (3), 100 (3), 106, 122 (3)	18 (3), 35

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Nr.	Structure	Order	Length	Maximal Subgroups	Minimal Overgroups
73	$Q_8 : 3$	24	1980	116, 134 (4)	19, 34, 52
74	$Q_8 : 3$	24	1980	115, 134 (4)	20, 34, 51
75	$2^2 : S_3$	24	1980	105, 123 (3), 130 (4)	27, 33, 54
76	$2^2 : S_3$	24	1980	105, 124 (3), 130 (4)	28, 31, 54
77	$A_4 \times 2$	24	1980	105, 125, 135 (4)	29, 32, 54
78	$A_4 \times 2$	24	1980	104, 126, 134 (4)	34, 53
79	$A_4 \times 2$	24	3960	107, 122, 134 (4)	34, 35
80	$D_{10} \cdot 2$	20	2376	110, 137 (5)	6, 25 (2), 28, 56
81	$D_{10} \cdot 2$	20	2376	110, 138 (5)	7, 26 (2), 27, 56
82	D_{20}	20	2376	109, 111, 110, 139 (5)	8, 10, 29, 56
83	$S_3 \times 3$	18	5280	112, 133, 134 (3)	35, 50
84	$3 : S_3$	18	220	114, 132 (12)	48 (4), 58 (3), 60 (6)
85	$S_3 \times 3$	18	2640	113, 132, 134 (3)	48, 50, 59
86	$3 : S_3$	18	220	113, 131 (12)	49 (4), 57 (3), 59 (6)
87	$S_3 \times 3$	18	2640	114, 131, 134 (3)	49, 50, 60
88	$Q_8 : 2$	16	2970	120, 119, 115	6, 23 (2), 51 (2), 63
89	$Q_8 : 2$	16	2970	118, 117, 116	7, 24 (2), 52 (2), 62
90	D_{16}	16	2970	118, 117, 127	8, 62
91	D_{16}	16	2970	120, 119, 128	10, 63
92	$D_8 \times 2$	16	1485	126 (2), 118 (2), 120 (2), 129	53 (4), 62, 63, 65
93	$D_8 \times 2$	16	2970	125, 121, 123, 128, 124, 122, 127	54 (2), 67
94	4^2	16	495	129 (3)	55, 64 (3), 65, 66 (3)
95	$8 : 2$	16	1485	129, 117 (2)	62, 64, 68
96	$D_8 : 2$	16	1485	123 (2), 121 (2), 129, 116, 127	62, 66, 67
97	$8 : 2$	16	1485	129, 119 (2)	63, 66, 68
98	$D_8 : 2$	16	1485	124 (2), 121 (2), 129, 115, 128	63, 64, 67
99	$D_8 \times 2$	16	1485	126, 123 (2), 125, 124 (2), 129	65, 67, 68
100	D_{12}	12	3960	135, 130 (2), 140 (3)	12 (2), 54 (2), 72
101	D_{12}	12	7920	132, 134, 131, 141 (3)	13, 25, 26, 51, 52, 53, 59, 60
102	D_{12}	12	3960	135, 130, 133, 139 (3)	27, 28, 29, 72
103	A_4	12	2640	140, 143 (4)	43 (3), 61
104	A_4	12	1980	141, 144 (4)	44 (2), 45 (2), 70, 71, 78
105	A_4	12	1980	142, 143 (4)	46, 55, 75, 76, 77
106	2×6	12	1320	135 (3), 140	61, 72
107	A_4	12	1320	140, 144 (4)	61, 79 (3)
108	11	11	1728	147	47

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Nr.	Structure	Order	Length	Maximal Subgroups	Minimal Overgroups
109	D_{10}	10	2376	136, 145 (5)	43 (4), 82
110	D_{10}	10	2376	136, 146 (5)	44 (2), 45 (2), 46, 80, 81, 82
111	10	10	2376	136, 145	82
112	3^2	9	1760	144, 143 (3)	61 (3), 69, 83 (3)
113	3^2	9	220	144 (4)	69 (4), 85 (12), 86
114	3^2	9	220	144 (4)	69 (4), 84, 87 (12)
115	Q_8	8	495	137 (3)	36 (4), 74 (4), 88 (6), 98 (3)
116	Q_8	8	495	138 (3)	37 (4), 73 (4), 89 (6), 96 (3)
117	8	8	2970	138	38 (2), 89, 90, 95
118	D_8	8	2970	141 (2), 138	39 (2), 71 (2), 89, 90, 92
119	8	8	2970	137	40 (2), 88, 91, 97
120	D_8	8	2970	141 (2), 137	41 (2), 70 (2), 88, 91, 92
121	2×4	8	2970	138, 137, 139	56 (4), 93, 96, 98
122	2^3	8	990	139 (3), 140 (4)	72 (4), 79 (4), 93 (3)
123	D_8	8	2970	142, 139, 138	75 (2), 93, 96, 99
124	D_8	8	2970	142, 139, 137	76 (2), 93, 98, 99
125	2^3	8	495	139 (6), 142	77 (4), 93 (6), 99 (3)
126	2^3	8	495	141 (4), 142 (3)	78 (4), 92 (6), 99 (3)
127	D_8	8	1485	139 (2), 138	90 (2), 93 (2), 96
128	D_8	8	1485	139 (2), 137	91 (2), 93 (2), 98
129	2×4	8	1485	142, 137, 138	92, 94, 95, 96, 97, 98, 99
130	S_3	6	3960	143, 145 (3)	43 (4), 75 (2), 76 (2), 100 (2), 102
131	S_3	6	2640	144, 146 (3)	44 (3), 70 (3), 86, 87, 101 (3)
132	S_3	6	2640	144, 146 (3)	45 (3), 71 (3), 84, 85, 101 (3)
133	S_3	6	1320	143, 146 (3)	46 (3), 83 (4), 102 (3)
134	6	6	7920	144, 146	73, 74, 78, 79 (2), 83 (2), 85, 87, 101
135	6	6	3960	143, 145	77 (2), 100, 102, 106
136	5	5	2376	147	47 (8), 109, 110, 111
137	4	4	1485	146	57 (4), 80 (8), 115, 119 (2), 120 (2), 121 (2), 124 (2), 128, 129

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Nr.	Structure	Order	Length	Maximal Subgroups	Minimal Overgroups
138	4	4	1485	146	58 (4), 81 (8), 116, 117 (2), 118 (2), 121 (2), 123 (2), 127, 129
139	2^2	4	2970	146, 145 (2)	82 (4), 102 (4), 121, 122, 123, 124, 125, 127, 128
140	2^2	4	1320	145 (3)	100 (9), 103 (2), 106, 107, 122 (3)
141	2^2	4	1980	146 (3)	101 (12), 104, 118 (3), 120 (3), 126
142	2^2	4	495	146 (3)	105 (4), 123 (6), 124 (6), 125, 126 (3), 129 (3)
143	3	3	1320	147	103 (8), 105 (6), 112 (4), 130 (3), 133, 135 (3)
144	3	3	880	147	104 (9), 107 (6), 112 (2), 113, 114, 131 (3), 132 (3), 134 (9)
145	2	2	396	147	109 (30), 111 (6), 130 (30), 135 (10), 139 (15), 140 (10)
146	2	2	495	147	110 (24), 131 (16), 132 (16), 133 (8), 134 (16), 137 (3), 138 (3), 139 (6), 141 (12), 142 (3)
147	1	1	1		108 (1728), 136 (2376), 143 (1320), 144 (880), 145 (396), 146 (495)

TABLE 1. Subgroup lattice of M_{12}

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