

THE SUBGROUP LATTICE OF A_9

THOMAS CONNOR AND DIMITRI LEEMANS

| Nr. | Structure | Order | Length | Maximal Subgroups | Minimal Overgroups |
|-----|-------------------------------|--------|--------|---|--------------------|
| 1 | A_9 | 181440 | 1 | 2 (9), 3 (36), 5 (84), 6 (120), 7 (120), 8 (126), 14 (280), 30 (840) | |
| 2 | A_8 | 20160 | 9 | 4 (8), 9 (15), 10 (15), 12 (28), 15 (35), 19 (56) | 1 |
| 3 | S_7 | 5040 | 36 | 4, 12 (7), 26 (21), 42 (35), 91 (120) | 1 |
| 4 | A_7 | 2520 | 36 | 20 (7), 37 (30), 45 (21), 62 (35) | 2 (2), 3 |
| 5 | $A_6 : S_3$ | 2160 | 84 | 11, 12 (3), 21 (6), 19 (6), 31 (10), 42 (15), 43 (15) | 1 |
| 6 | $L_2(8) : 3$ | 1512 | 120 | 16, 38 (9), 77 (28), 91 (36) | 1 |
| 7 | $L_2(8) : 3$ | 1512 | 120 | 17, 39 (9), 78 (28), 91 (36) | 1 |
| 8 | $A_5 \times 2 \times 2 : S_3$ | 1440 | 126 | 13, 18 (3), 19 (4), 23 (5), 27 (6), 42 (10) | 1 |
| 9 | $2^3 : L_2(7)$ | 1344 | 135 | 32 (7), 33 (7), 37 (8), 40 (8), 39 (8) | 2 |
| 10 | $2^3 : L_2(7)$ | 1344 | 135 | 34 (7), 32 (7), 37 (8), 40 (8), 38 (8) | 2 |
| 11 | $A_6 \times 3$ | 1080 | 84 | 20, 35 (6), 36 (6), 49 (10), 63 (15), 64 (15) | 5 |
| 12 | S_6 | 720 | 252 | 20, 46 (6), 45 (6), 65 (10), 82 (15), 83 (15) | 2, 3, 5 |
| 13 | $A_5 \times 2 \times 2 : 3$ | 720 | 126 | 28, 36 (4), 44 (5), 47 (6), 66 (10) | 8 |
| 14 | $3 : S_3 : S_3 : S_3$ | 648 | 280 | 22, 31 (3), 41 (4), 114 (27) | 1 |
| 15 | $2^4 : S_3 : S_3$ | 576 | 315 | 23, 24, 25, 34 (3), 33 (3), 95 (16) | 2 |

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| Nr. | Structure | Order | Length | Maximal Subgroups | Minimal Overgroups |
| 16 | $L_2(8)$ | 504 | 120 | 75 (9), 141 (28), 162 (36) | 6 |
| 17 | $L_2(8)$ | 504 | 120 | 76 (9), 142 (28), 162 (36) | 7 |
| 18 | $A_5 \times 2 \times 2 : 2$ | 480 | 378 | 29, 26, 28, 52 (5), 61 (6), 84 (10) | 8 |
| 19 | $A_5 : S_3$ | 360 | 504 | 36, 45 (3), 62 (5), 71 (6), 96 (10) | 2, 5, 8 |
| 20 | A_6 | 360 | 84 | 72 (6), 73 (6), 97 (10), 115 (15), 116 (15) | 4 (3), 11, 12 (3) |
| 21 | $A_5 : S_3$ | 360 | 504 | 35, 46 (3), 67 (5), 71 (6), 98 (10) | 5 |
| 22 | $3 : S_3 : S_3 : 3$ | 324 | 280 | 50, 60 (4), 163 (27) | 14 |
| 23 | $2^4 : 3 : S_3$ | 288 | 315 | 44, 53 (3), 52 (6), 54 (3), 62 (8) | 8 (2), 15 |
| 24 | $2^4 : 3 : S_3$ | 288 | 315 | 44, 55, 56 (3), 143 (16) | 15 |
| 25 | $2^4 : 3 : S_3$ | 288 | 315 | 44, 57, 58 (3), 143 (16) | 15 |
| 26 | $S_5 \times 2$ | 240 | 378 | 45 (2), 48, 82 (5), 92 (6), 117 (10) | 3 (2), 18 |
| 27 | $D_{20} \times 2 \cdot S_3$ | 240 | 756 | 47, 61 (3), 71 (4), 85 (5) | 8 |
| 28 | $A_5 \times 2 \times 2$ | 240 | 126 | 48 (3), 86 (5), 93 (6), 118 (10) | 13, 18 (3) |
| 29 | $A_5 \times 2 \cdot 2$ | 240 | 378 | 48, 85 (5), 94 (6), 119 (10) | 18 |
| 30 | $M_9 : 3$ | 216 | 840 | 68, 79 (4), 120 (9) | 1 |
| 31 | $3 : S_3 \cdot 2 : S_3$ | 216 | 840 | 51, 50, 49, 65 (3), 121 (9) | 5, 14 |
| 32 | $2^{1+4} : S_3$ | 192 | 945 | 59, 70 (3), 83 (4), 87 (4), 88 (4) | 9, 10 |
| 33 | $D_8 \times 2 : 2 : S_3$ | 192 | 945 | 58, 53, 55, 70 (3), 87 (4) | 9, 15 |
| 34 | $D_8 \times 2 : 2 : S_3$ | 192 | 945 | 57, 54, 56, 70 (3), 88 (4) | 10, 15 |
| 35 | $A_5 \times 3$ | 180 | 504 | 73, 99 (5), 109 (6), 144 (10) | 11, 21 |
| 36 | $A_5 \times 3$ | 180 | 504 | 72, 100 (5), 109 (6), 145 (10) | 11, 13, 19 |
| 37 | $L_2(7)$ | 168 | 1080 | 122 (7), 115 (7), 135 (8) | 4, 9, 10 |

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| Nr. | Structure | Order | Length | Maximal Subgroups | Minimal Overgroups |
| 38 | $2^3 : 7 : 3$ | 168 | 1080 | 75, 123 (7), 135 (8) | 6, 10 |
| 39 | $2^3 : 7 : 3$ | 168 | 1080 | 76, 124 (7), 135 (8) | 7, 9 |
| 40 | $L_2(7)$ | 168 | 1080 | 125 (7), 126 (7), 135 (8) | 9, 10 |
| 41 | $3^{1+2} : S_3$ | 162 | 1120 | 60, 80 (3), 77 (3), 79 (3), 78 (3) | 14 |
| 42 | $S_3 \times 2^2 : S_3$ | 144 | 1260 | 63, 62, 66, 84 (3), 82 (3), 96 (4) | 3, 5, 8 |
| 43 | $S_3 \times 2^2 : S_3$ | 144 | 1260 | 69, 64, 67, 84 (3), 83 (3), 98 (4) | 5 |
| 44 | $2^4 : 3 : 3$ | 144 | 315 | 86 (2), 89, 90, 100 (8) | 13 (2), 23, 24, 25 |
| 45 | S_5 | 120 | 756 | 72, 116 (5), 136 (6), 164 (10) | 4, 12 (2), 19 (2), 26 |
| 46 | $A_5 : 2$ | 120 | 1512 | 73, 127 (5), 136 (6), 165 (10) | 12, 21 |
| 47 | $2 \times 10 : 3 : 2$ | 120 | 756 | 74, 93, 109 (4), 128 (5) | 13, 27 |
| 48 | $A_5 \times 2$ | 120 | 378 | 72, 128 (5), 137 (6), 166 (10) | 26, 28, 29 |
| 49 | $3^2 : S_3 \cdot 2$ | 108 | 840 | 81, 97, 167 (9) | 11, 31 |
| 50 | $3 : S_3 : S_3$ | 108 | 280 | 81 (3), 96 (9) | 22, 31 (3) |
| 51 | $3 : S_3 : S_3$ | 108 | 840 | 80 (2), 81, 98 (6), 95 (3) | 31 |
| 52 | $2^4 : S_3$ | 96 | 1890 | 86, 82, 85, 102 (3), 129 (4) | 18, 23 |
| 53 | $2^4 : S_3$ | 96 | 945 | 89, 102 (3), 122 (8), 130 (4) | 23, 33 |
| 54 | $2^4 : S_3$ | 96 | 945 | 90, 102 (3), 122 (8), 131 (4) | 23, 34 |
| 55 | $2^4 : S_3$ | 96 | 315 | 89, 103 (3), 125 (12) | 24, 33 (3) |
| 56 | $2^4 : 3 : 2$ | 96 | 945 | 90, 103, 123 (4) | 24, 34 |
| 57 | $2^4 : S_3$ | 96 | 315 | 90, 104 (3), 126 (12) | 25, 34 (3) |
| 58 | $2^4 : 3 : 2$ | 96 | 945 | 89, 104, 124 (4) | 25, 33 |
| 59 | $2^{1+4} : 3$ | 96 | 945 | 105, 123 (4), 132 (4), 120 (8), 124 (4) | 32 |
| 60 | $3^{1+2} : 3$ | 81 | 1120 | 110, 111, 112, 113 | 22, 41 |
| 61 | $D_{10} \cdot 2 \times 2 : 2$ | 80 | 2268 | 94, 93, 92, 151 (5) | 18, 27 |
| 62 | $A_4 : S_3$ | 72 | 1260 | 100, 122 (6), 116 (3), 129 (3), 146 (4) | 4, 19 (2), 23 (2), 42 |
| 63 | $2 \times 6 : S_3$ | 72 | 1260 | 100, 116, 133 (3), 145 (4) | 11, 42 |
| 64 | $2 \times 6 : S_3$ | 72 | 1260 | 99, 115, 133 (3), 144 (4) | 11, 43 |

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| Nr. | Structure | Order | Length | Maximal Subgroups | Minimal Overgroups |
| 65 | $S_3 \times S_3 : 2$ | 72 | 2520 | 97, 95, 96, 186 (9) | 12, 31 |
| 66 | $A_4 \times S_3$ | 72 | 1260 | 100, 118, 128 (3), 145 (4) | 13, 42 |
| 67 | $A_4 : S_3$ | 72 | 1260 | 99, 114 (6), 127 (3), 134 (3), 147 (4) | 21 (2), 43 |
| 68 | M_9 | 72 | 840 | 101 (3), 187 (9) | 30 |
| 69 | $A_4 \times S_3$ | 72 | 1260 | 99, 117, 132 (3), 148 (4) | 43 |
| 70 | $2^{1+4} : 2$ | 64 | 2835 | 102, 106, 104, 107, 103, 108, 105 | 32, 33, 34 |
| 71 | $D_{10} \cdot S_3$ | 60 | 3024 | 109, 136 (3), 168 (5) | 19, 21, 27 |
| 72 | A_5 | 60 | 126 | 169 (5), 177 (6), 202 (10) | 20 (4), 36 (4), 45 (6), 48 (3) |
| 73 | A_5 | 60 | 504 | 170 (5), 177 (6), 203 (10) | 20, 35, 46 (3) |
| 74 | $2 \times 10 : 3$ | 60 | 756 | 138, 161 (4), 169 | 47 |
| 75 | $2^3 : 7$ | 56 | 1080 | 188, 201 (8) | 16, 38 |
| 76 | $2^3 : 7$ | 56 | 1080 | 189, 201 (8) | 17, 39 |
| 77 | $3^{1+2} : 2$ | 54 | 3360 | 113, 141, 149 (3) | 6, 41 |
| 78 | $3^{1+2} : 2$ | 54 | 3360 | 112, 142, 149 (3) | 7, 41 |
| 79 | $3^{1+2} : 2$ | 54 | 3360 | 111, 150, 149 (3) | 30, 41 |
| 80 | $3^2 : S_3$ | 54 | 1680 | 110, 147, 149 (6), 143 (3), 148 (3) | 41 (2), 51 |
| 81 | $3^2 : S_3$ | 54 | 840 | 110, 146, 145 (6), 144 (6) | 49, 50, 51 |
| 82 | $2^3 : S_3$ | 48 | 1890 | 116 (2), 128, 152 (3), 164 (4) | 12 (2), 26, 42 (2), 52 |
| 83 | $2^3 : S_3$ | 48 | 3780 | 127, 132, 115, 152 (3), 165 (4) | 12, 32, 43 |
| 84 | $D_8 \times S_3$ | 48 | 3780 | 134, 133, 118, 117, 129, 119, 121, 152 (3) | 18, 42, 43 |
| 85 | $2^3 \cdot S_3$ | 48 | 1890 | 128, 151 (3), 168 (4) | 27 (2), 29, 52 |
| 86 | $A_4 \times 2 \times 2$ | 48 | 630 | 128 (3), 153, 171 (4) | 28, 44, 52 (3) |
| 87 | $2^3 : S_3$ | 48 | 3780 | 124, 130, 125, 154 (3), 165 (4) | 32, 33 |
| 88 | $2^3 : S_3$ | 48 | 3780 | 123, 131, 126, 155 (3), 165 (4) | 32, 34 |
| 89 | $2^4 : 3$ | 48 | 315 | 153, 172 (8), 173 (12) | 44, 53 (3), 55, 58 (3) |
| 90 | $2^4 : 3$ | 48 | 315 | 153, 172 (8), 174 (12) | 44, 54 (3), 56 (3), 57 |
| 91 | $7 : 3 : 2$ | 42 | 4320 | 135, 162, 204 (7) | 3, 6, 7 |
| 92 | $D_{10} \cdot 2 \times 2$ | 40 | 2268 | 136 (2), 137, 190 (5) | 26, 61 |
| 93 | $D_{20} \times 2$ | 40 | 756 | 137 (3), 139 (3), 138, 191 (5) | 28, 47, 61 (3) |

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| Nr. | Structure | Order | Length | Maximal Subgroups | Minimal Overgroups |
| 94 | $D_{10} \cdot 2 \times 2$ | 40 | 2268 | 140 (2), 137, 192 (5) | 29, 61 |
| 95 | $S_3 \times S_3$ | 36 | 2520 | 143 (2), 146, 165 (6) | 15 (2), 51, 65 |
| 96 | $S_3 \times S_3$ | 36 | 2520 | 146, 145 (2), 164 (6) | 19 (2), 42 (2), 50, 65 |
| 97 | $3 : S_3 \cdot 2$ | 36 | 840 | 146, 210 (9) | 20, 49, 65 (3) |
| 98 | $S_3 \times S_3$ | 36 | 5040 | 147, 144, 148, 175 (3), 165 (3) | 21, 43, 51 |
| 99 | $A_4 \times 3$ | 36 | 1260 | 163 (2), 170, 176, 180 (4) | 35 (2), 64, 67, 69 |
| 100 | $A_4 \times 3$ | 36 | 1260 | 172 (2), 171, 169, 181 (4) | 36 (2), 44 (2), 62, 63, 66 |
| 101 | $3 : S_3 \cdot 2$ | 36 | 2520 | 150, 211 (9) | 68 |
| 102 | $2 \times 4 : 2 : 2$ | 32 | 2835 | 152 (2), 156, 157, 151 (2), 153 | 52 (2), 53, 54, 70 |
| 103 | $2 \times 4 : 2 : 2$ | 32 | 945 | 153, 158 (3), 155 (3) | 55, 56, 70 (3) |
| 104 | $2 \times 4 : 2 : 2$ | 32 | 945 | 153, 154 (3), 159 (3) | 57, 58, 70 (3) |
| 105 | 2^{1+4} | 32 | 945 | 157 (3), 154 (3), 160 (6), 155 (3) | 59, 70 (3) |
| 106 | $2^3 \cdot 2 : 2$ | 32 | 2835 | 156, 154, 158 | 70 |
| 107 | $2 \times 4 : 2 : 2$ | 32 | 2835 | 157, 159, 158 | 70 |
| 108 | $D_8 \times 2 \cdot 2$ | 32 | 2835 | 156, 159, 155 | 70 |
| 109 | $15 : 2$ | 30 | 3024 | 161, 177, 205 (5) | 35, 36, 47, 71 |
| 110 | 3^3 | 27 | 280 | 181 (3), 182 (4), 180 (6) | 60 (4), 80 (6), 81 (3) |
| 111 | 3^{1+2} | 27 | 1120 | 182, 183 (3) | 60, 79 (3) |
| 112 | 3^{1+2} | 27 | 1120 | 182, 184 (3) | 60, 78 (3) |
| 113 | 3^{1+2} | 27 | 1120 | 182, 185 (3) | 60, 77 (3) |
| 114 | $2^2 : S_3$ | 24 | 7560 | 163, 186 (3), 206 (4) | 14, 67 |
| 115 | $2^2 : S_3$ | 24 | 1260 | 170, 193 (3), 203 (4) | 20, 37 (6), 64, 83 (3) |
| 116 | $2^2 : S_3$ | 24 | 1260 | 169, 193 (3), 202 (4) | 20, 45 (3), 62 (3), 63, 82 (3) |
| 117 | $S_3 \times 2^2$ | 24 | 1260 | 164 (3), 166 (3), 176, 194 (3) | 26 (3), 69, 84 (3) |
| 118 | $S_3 \times 2^2$ | 24 | 1260 | 166 (3), 175 (3), 171, 191 (3) | 28, 66, 84 (3) |
| 119 | $S_3 \times 4$ | 24 | 3780 | 168, 167, 166, 190 (3) | 29, 84 |
| 120 | $Q_8 : 3$ | 24 | 7560 | 187, 204 (4) | 30, 59 |
| 121 | D_{24} | 24 | 3780 | 167, 175, 164, 186 (3) | 31 (2), 84 |
| 122 | $2^2 : S_3$ | 24 | 7560 | 172, 193 (3), 203 (4) | 37, 53, 54, 62 |
| 123 | $A_4 \times 2$ | 24 | 3780 | 174, 188, 204 (4) | 38 (2), 56, 59, 88 |
| 124 | $A_4 \times 2$ | 24 | 3780 | 173, 189, 204 (4) | 39 (2), 58, 59, 87 |
| 125 | $2^2 : S_3$ | 24 | 3780 | 173, 195 (3), 207 (4) | 40 (2), 55, 87 |
| 126 | $2^2 : S_3$ | 24 | 3780 | 174, 195 (3), 207 (4) | 40 (2), 57, 88 |
| 127 | $2^2 : S_3$ | 24 | 3780 | 170, 186 (3), 207 (4) | 46 (2), 67, 83 |

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| Nr. | Structure | Order | Length | Maximal Subgroups | Minimal Overgroups |
| 128 | $A_4 \times 2$ | 24 | 1890 | 169, 191, 205 (4) | 47 (2), 48, 66 (2), 82, 85, 86 |
| 129 | $2^2 : S_3$ | 24 | 3780 | 168, 171, 164, 193 (3) | 52 (2), 62, 84 |
| 130 | $2^2 : S_3$ | 24 | 3780 | 173, 196 (3), 203 (4) | 53, 87 |
| 131 | $2^2 : S_3$ | 24 | 3780 | 174, 197 (3), 203 (4) | 54, 88 |
| 132 | $A_4 \times 2$ | 24 | 3780 | 170, 194, 204 (4) | 59, 69, 83 |
| 133 | $D_8 \times 3$ | 24 | 3780 | 167, 171, 176, 193 | 63, 64, 84 |
| 134 | $2^2 : S_3$ | 24 | 3780 | 175, 176, 168, 186 (3) | 67, 84 |
| 135 | $7 : 3$ | 21 | 4320 | 201, 218 (7) | 37 (2), 38 (2), 39 (2), 40 (2), 91 |
| 136 | $D_{10} \cdot 2$ | 20 | 4536 | 177, 210 (5) | 45, 46 (2), 71 (2), 92 |
| 137 | D_{20} | 20 | 2268 | 178, 177, 179, 212 (5) | 48, 92, 93, 94 |
| 138 | 2×10 | 20 | 756 | 179 (3), 213 | 74, 93 |
| 139 | D_{20} | 20 | 2268 | 178 (2), 179, 214 (5) | 93 |
| 140 | $D_{10} \cdot 2$ | 20 | 4536 | 178, 211 (5) | 94 |
| 141 | D_{18} | 18 | 3360 | 185, 206 (3) | 16, 77 |
| 142 | D_{18} | 18 | 3360 | 184, 206 (3) | 17, 78 |
| 143 | $S_3 \times 3$ | 18 | 5040 | 181, 207, 204 (3) | 24, 25, 80, 95 |
| 144 | $S_3 \times 3$ | 18 | 5040 | 180, 203, 205 (3) | 35, 64, 81, 98 |
| 145 | $S_3 \times 3$ | 18 | 5040 | 181, 202, 205 (3) | 36, 63, 66, 81, 96 |
| 146 | $3 : S_3$ | 18 | 840 | 181, 202 (6), 203 (6) | 62 (6), 81, 95 (3), 96 (3), 97 |
| 147 | $3 : S_3$ | 18 | 1680 | 180, 208 (3), 206 (6), 207 (3) | 67 (3), 80, 98 (3) |
| 148 | $S_3 \times 3$ | 18 | 5040 | 180, 208, 204 (3) | 69, 80, 98 |
| 149 | $S_3 \times 3$ | 18 | 10080 | 182, 206, 204 (3) | 77, 78, 79, 80 |
| 150 | $3 : S_3$ | 18 | 840 | 183, 206 (12) | 79 (4), 101 (3) |
| 151 | $2^3 \cdot 2$ | 16 | 5670 | 190, 192, 191 | 61 (2), 85, 102 |
| 152 | $D_8 \times 2$ | 16 | 5670 | 194, 190, 186 (2), 193 (2), 191 | 82, 83 (2), 84 (2), 102 |
| 153 | 2^4 | 16 | 315 | 191 (6), 198 (9) | 86 (2), 89, 90, 102 (9), 103 (3), 104 (3) |
| 154 | $D_8 \times 2$ | 16 | 2835 | 196 (2), 199, 198, 195 (2), 189 | 87 (4), 104, 105, 106 |
| 155 | $D_8 \times 2$ | 16 | 2835 | 198, 195 (2), 197 (2), 200, 188 | 88 (4), 103, 105, 108 |
| 156 | $2 \times 4 : 2$ | 16 | 2835 | 190 (2), 198 | 102, 106, 108 |
| 157 | $D_8 \times 2$ | 16 | 2835 | 194, 196 (2), 192, 197 (2), 198 | 102, 105, 107 |
| 158 | $2 \times 4 : 2$ | 16 | 2835 | 198, 200 (2) | 103, 106, 107 |
| 159 | $2 \times 4 : 2$ | 16 | 2835 | 198, 199 (2) | 104, 107, 108 |
| 160 | $D_8 : 2$ | 16 | 5670 | 192, 199, 196, 187, 197, 200, 195 | 105 |

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| Nr. | Structure | Order | Length | Maximal Subgroups | Minimal Overgroups |
| 161 | 15 | 15 | 3024 | 209, 219 | 74, 109 |
| 162 | D_{14} | 14 | 4320 | 201, 221 (7) | 16, 17, 91 |
| 163 | A_4 | 12 | 2520 | 215, 220 (4) | 22 (3), 99, 114 (3) |
| 164 | D_{12} | 12 | 3780 | 202 (2), 205, 215 (3) | 45 (2), 82 (2), 96 (4), 117, 121, 129 |
| 165 | D_{12} | 12 | 15120 | 204, 207, 203, 214 (3) | 46, 83, 87, 88, 95, 98 |
| 166 | D_{12} | 12 | 3780 | 208, 202, 205, 212 (3) | 48, 117, 118, 119 |
| 167 | 12 | 12 | 3780 | 205, 210 | 49 (2), 119, 121, 133 |
| 168 | Q_{12} | 12 | 3780 | 205, 210 (3) | 71 (4), 85 (2), 119, 129, 134 |
| 169 | A_4 | 12 | 126 | 213, 219 (4) | 72 (5), 74 (6), 100 (10), 116 (10), 128 (15) |
| 170 | A_4 | 12 | 1260 | 215, 218 (4) | 73 (2), 99, 115, 127 (3), 132 (3) |
| 171 | 2×6 | 12 | 1260 | 205 (3), 213 | 86 (2), 100, 118, 129 (3), 133 (3) |
| 172 | A_4 | 12 | 2520 | 213, 218 (4) | 89, 90, 100, 122 (3) |
| 173 | A_4 | 12 | 3780 | 216, 218 (4) | 89, 124, 125, 130 |
| 174 | A_4 | 12 | 3780 | 217, 218 (4) | 90, 123, 126, 131 |
| 175 | D_{12} | 12 | 3780 | 208 (2), 205, 214 (3) | 98 (4), 118, 121, 134 |
| 176 | 2×6 | 12 | 1260 | 205 (3), 215 | 99, 117, 133 (3), 134 (3) |
| 177 | D_{10} | 10 | 756 | 209, 222 (5) | 72, 73 (4), 109 (4), 136 (6), 137 (3) |
| 178 | D_{10} | 10 | 2268 | 209, 221 (5) | 137, 139 (2), 140 (2) |
| 179 | 10 | 10 | 2268 | 209, 222 | 137, 138, 139 |
| 180 | 3^2 | 9 | 1680 | 219, 220 (2), 218 | 99 (3), 110, 144 (3), 147, 148 (3) |
| 181 | 3^2 | 9 | 840 | 219 (2), 218 (2) | 100 (6), 110, 143 (6), 145 (6), 146 |
| 182 | 3^2 | 9 | 1120 | 218 (3), 220 | 110, 111, 112, 113, 149 (9) |
| 183 | 3^2 | 9 | 840 | 220 (4) | 111 (4), 150 |
| 184 | 9 | 9 | 3360 | 220 | 112, 142 |
| 185 | 9 | 9 | 3360 | 220 | 113, 141 |
| 186 | D_8 | 8 | 11340 | 215, 210, 214 | 65 (2), 114 (2), 121, 127, 134, 152 |
| 187 | Q_8 | 8 | 1890 | 211 (3) | 68 (4), 120 (4), 160 (3) |
| 188 | 2^3 | 8 | 135 | 217 (7) | 75 (8), 123 (28), 155 (21) |

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| Nr. | Structure | Order | Length | Maximal Subgroups | Minimal Overgroups |
| 189 | 2^3 | 8 | 135 | 216 (7) | 76 (8), 124 (28), 154 (21) |
| 190 | 2×4 | 8 | 5670 | 212, 210 (2) | 92 (2), 119 (2), 151, 152, 156 |
| 191 | 2^3 | 8 | 1890 | 212 (3), 214 (3), 213 | 93 (2), 118 (2), 128, 151 (3), 152 (3), 153 |
| 192 | 2×4 | 8 | 2835 | 212, 211 (2) | 94 (4), 151 (2), 157, 160 (2) |
| 193 | D_8 | 8 | 3780 | 215, 210, 213 | 115, 116, 122 (6), 129 (3), 133, 152 (3) |
| 194 | 2^3 | 8 | 945 | 215 (4), 212 (3) | 117 (4), 132 (4), 152 (6), 157 (3) |
| 195 | D_8 | 8 | 5670 | 217, 211, 216 | 125 (2), 126 (2), 154, 155, 160 |
| 196 | D_8 | 8 | 5670 | 212, 211, 216 | 130 (2), 154, 157, 160 |
| 197 | D_8 | 8 | 5670 | 212, 211, 217 | 131 (2), 155, 157, 160 |
| 198 | 2^3 | 8 | 2835 | 214 (4), 217, 216, 212 | 153, 154, 155, 156, 157, 158, 159 |
| 199 | 2×4 | 8 | 2835 | 211 (2), 216 | 154, 159 (2), 160 (2) |
| 200 | 2×4 | 8 | 2835 | 211 (2), 217 | 155, 158 (2), 160 (2) |
| 201 | 7 | 7 | 4320 | 223 | 75 (2), 76 (2), 135, 162 |
| 202 | S_3 | 6 | 1260 | 219, 222 (3) | 72, 116 (4), 145 (4), 146 (4), 164 (6), 166 (3) |
| 203 | S_3 | 6 | 5040 | 218, 222 (3) | 73, 115, 122 (6), 130 (3), 131 (3), 144, 146, 165 (3) |
| 204 | 6 | 6 | 15120 | 218, 221 | 91 (2), 120 (2), 123, 124, 132, 143, 148, 149 (2), 165 |
| 205 | 6 | 6 | 3780 | 219, 222 | 109 (4), 128 (2), 144 (4), 145 (4), 164, 166, 167, 168, 171, 175, 176 |
| 206 | S_3 | 6 | 10080 | 220, 221 (3) | 114 (3), 141, 142, 147, 149, 150 |
| 207 | S_3 | 6 | 5040 | 218, 221 (3) | 125 (3), 126 (3), 127 (3), 143, 147, 165 (3) |
| 208 | S_3 | 6 | 1260 | 219, 221 (3) | 147 (4), 148 (4), 166 (3), 175 (6) |
| 209 | 5 | 5 | 756 | 223 | 161 (4), 177, 178 (3), 179 (3) |

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| <i>continued from previous page</i> | | | | | |
|-------------------------------------|-----------|-------|--------|-------------------|---|
| Nr. | Structure | Order | Length | Maximal Subgroups | Minimal Overgroups |
| 210 | 4 | 4 | 3780 | 222 | 97 (2), 136 (6), 167, 168 (3), 186 (3), 190 (3), 193 |
| 211 | 4 | 4 | 5670 | 221 | 101 (4), 140 (4), 187, 192, 195, 196, 197, 199, 200 |
| 212 | 2^2 | 4 | 2835 | 222 (2), 221 | 137 (4), 166 (4), 190 (2), 191 (2), 192, 194, 196 (2), 197 (2), 198 |
| 213 | 2^2 | 4 | 126 | 222 (3) | 138 (6), 169, 171 (10), 172 (20), 191 (15), 193 (30) |
| 214 | 2^2 | 4 | 5670 | 221 (2), 222 | 139 (2), 165 (8), 175 (2), 186 (2), 191, 198 (2) |
| 215 | 2^2 | 4 | 1260 | 222 (3) | 163 (2), 164 (9), 170, 176, 186 (9), 193 (3), 194 (3) |
| 216 | 2^2 | 4 | 945 | 221 (3) | 173 (4), 189, 195 (6), 196 (6), 198 (3), 199 (3) |
| 217 | 2^2 | 4 | 945 | 221 (3) | 174 (4), 188, 195 (6), 197 (6), 198 (3), 200 (3) |
| 218 | 3 | 3 | 1680 | 223 | 135 (18), 170 (3), 172 (6), 173 (9), 174 (9), 180, 181, 182 (2), 203 (3), 204 (9), 207 (3) |
| 219 | 3 | 3 | 84 | 223 | 161 (36), 169 (6), 180 (20), 181 (20), 202 (15), 205 (45), 208 (15) |
| 220 | 3 | 3 | 1120 | 223 | 163 (9), 180 (3), 182, 183 (3), 184 (3), 185 (3), 206 (9) |
| 221 | 2 | 2 | 945 | 223 | 162 (32), 178 (12), 204 (16), 206 (32), 207 (16), 208 (4), 211 (6), 212 (3), 214 (12), 216 (3), 217 (3) |

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| <i>continued from previous page</i> | | | | | |
|-------------------------------------|-----------|-------|--------|-------------------|--|
| Nr. | Structure | Order | Length | Maximal Subgroups | Minimal Overgroups |
| 222 | 2 | 2 | 378 | 223 | 177 (10), 179 (6), 202 (10), 203 (40), 205 (10), 210 (10), 212 (15), 213, 214 (15), 215 (10) |
| 223 | 1 | 1 | 1 | | 201 (4320), 209 (756), 218 (1680), 219 (84), 220 (1120), 221 (945), 222 (378) |

TABLE 1. Subgroup lattice of A_9

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