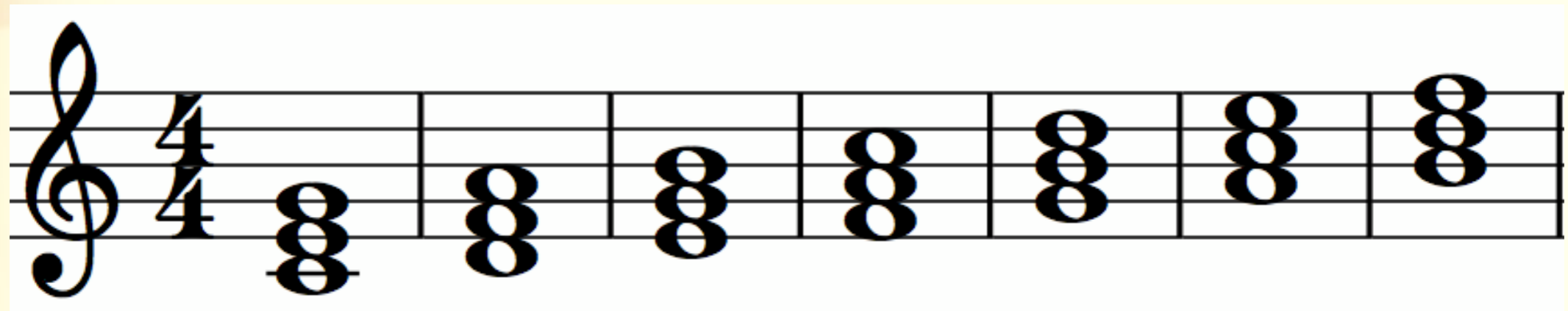
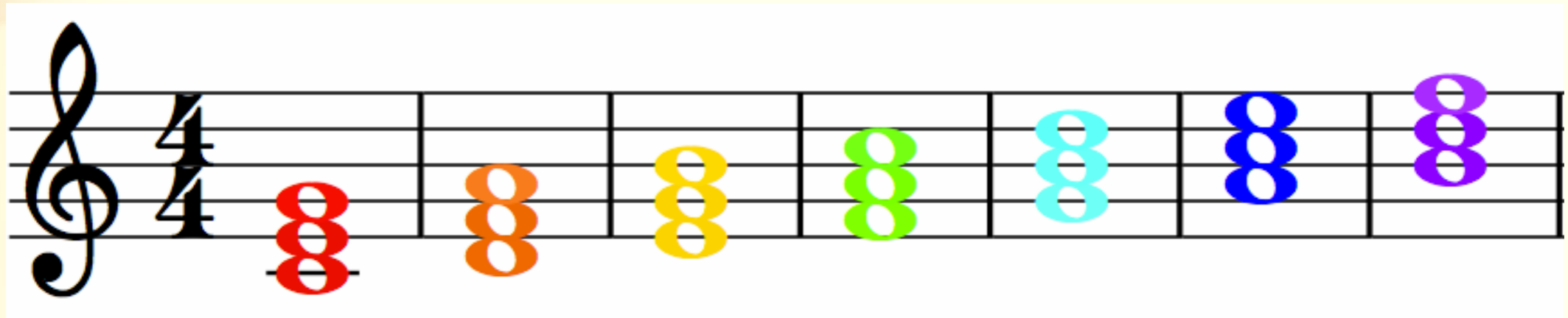
The background of the slide features a dark, almost black, field filled with numerous out-of-focus, glowing circles in shades of yellow and orange. These circles vary in size and brightness, creating a bokeh effect that is reminiscent of light reflecting off water or particles in a dark space. The overall aesthetic is warm and abstract.

# Triad colorings of triangulations on closed surfaces

Yumiko Ohno

Yokohama National University M1

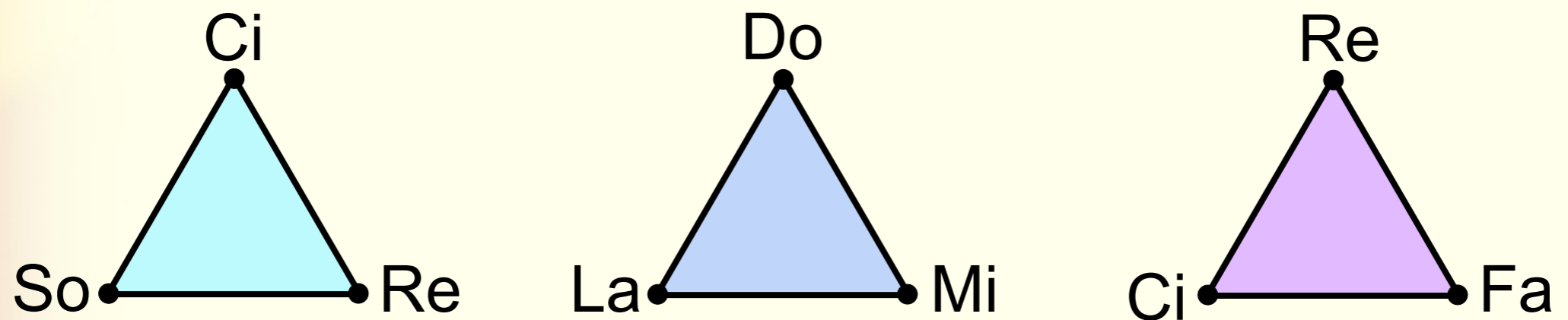
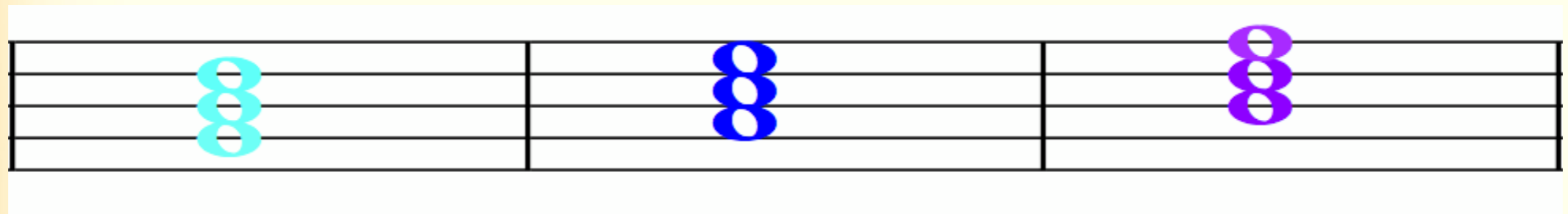
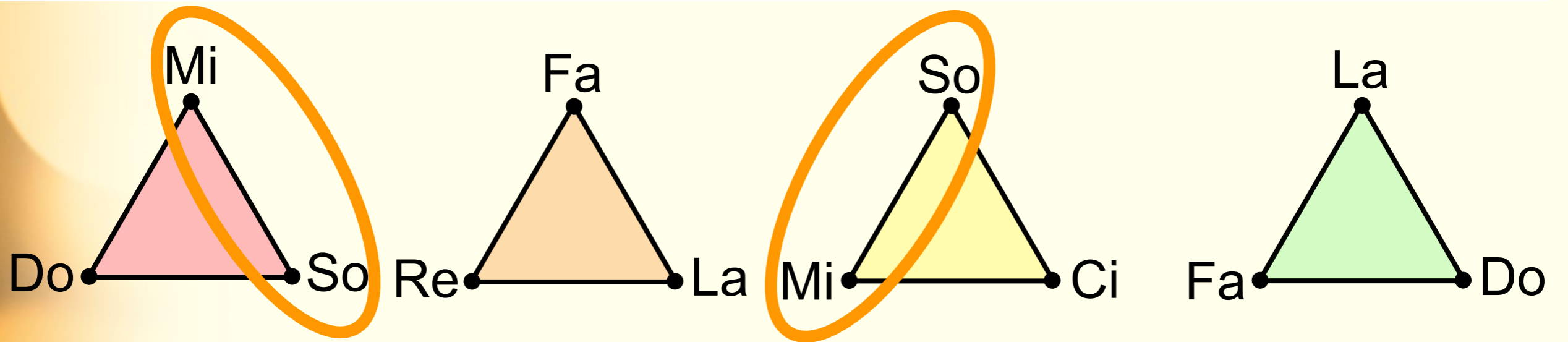
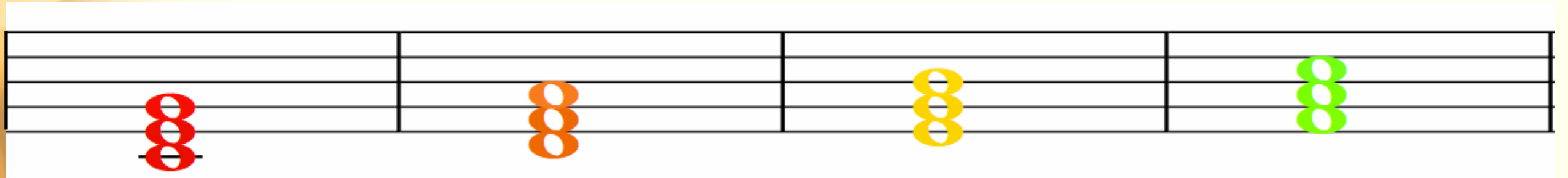


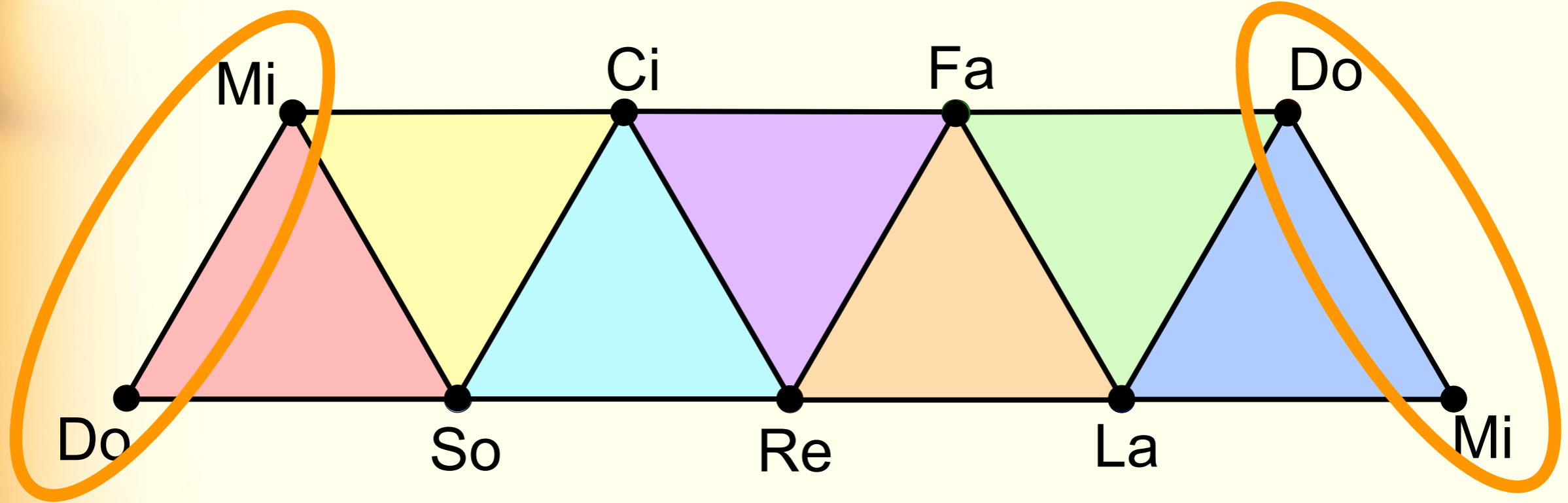
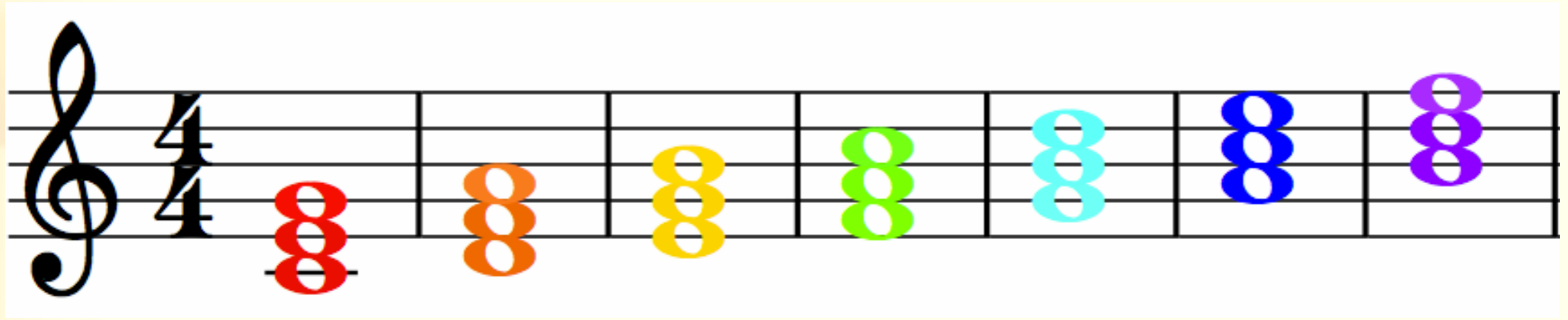


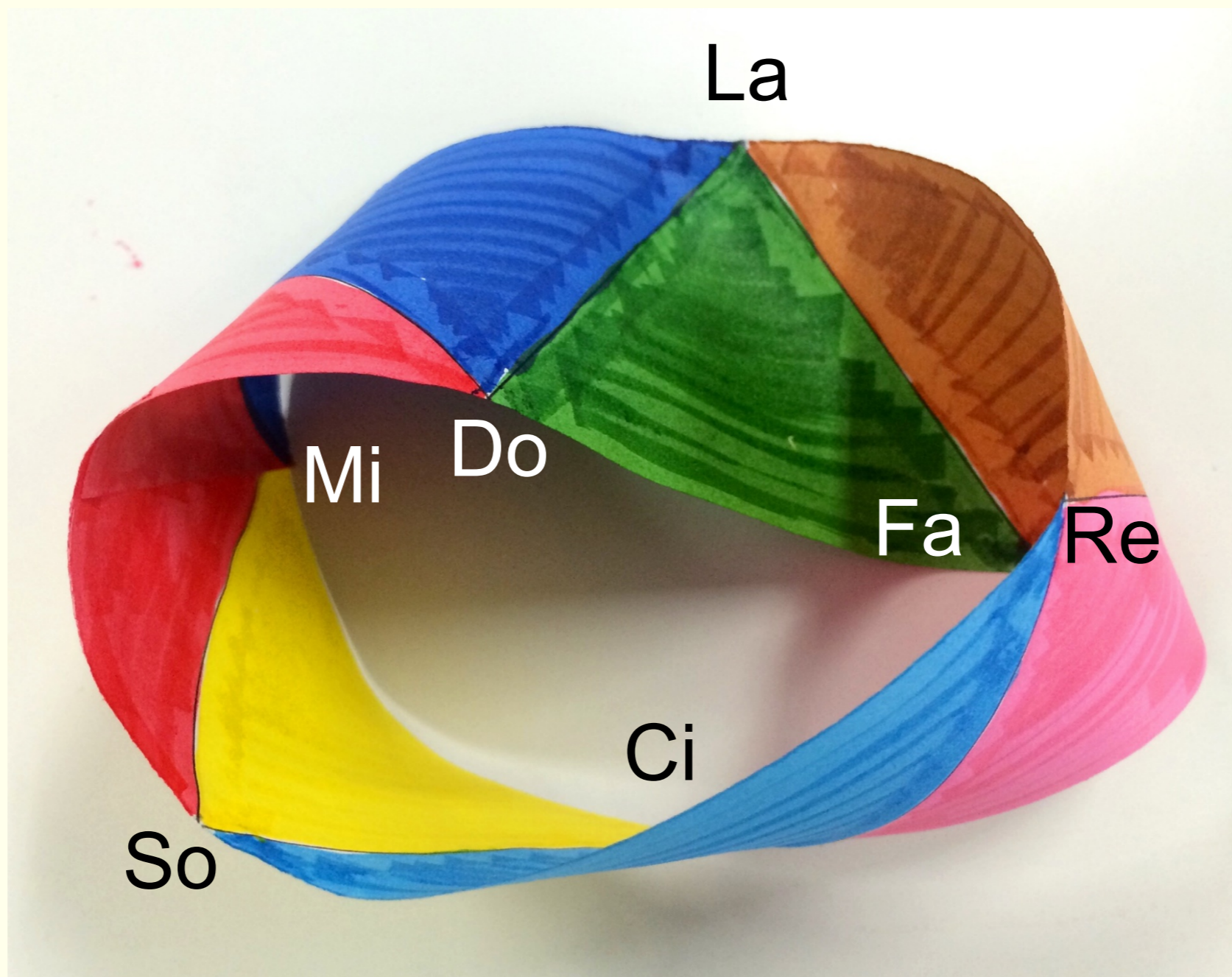
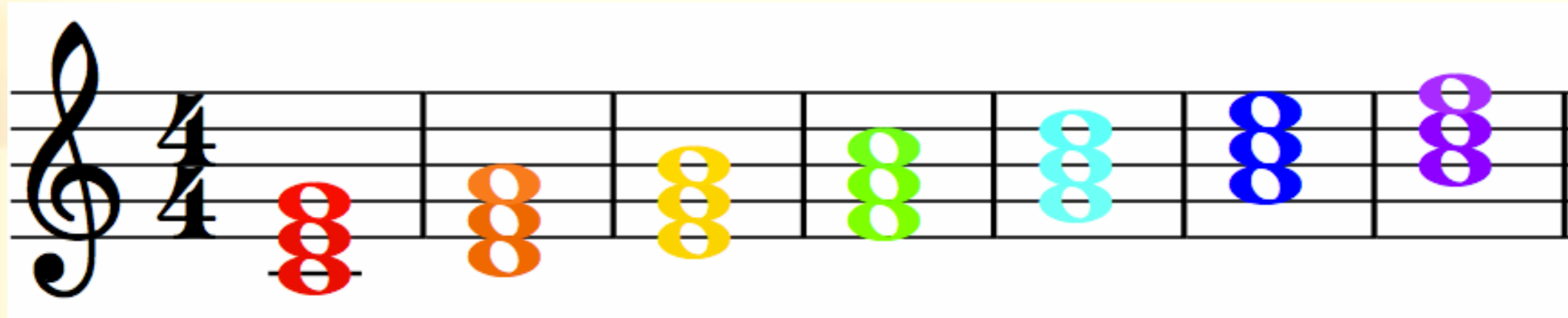
Do Mi So

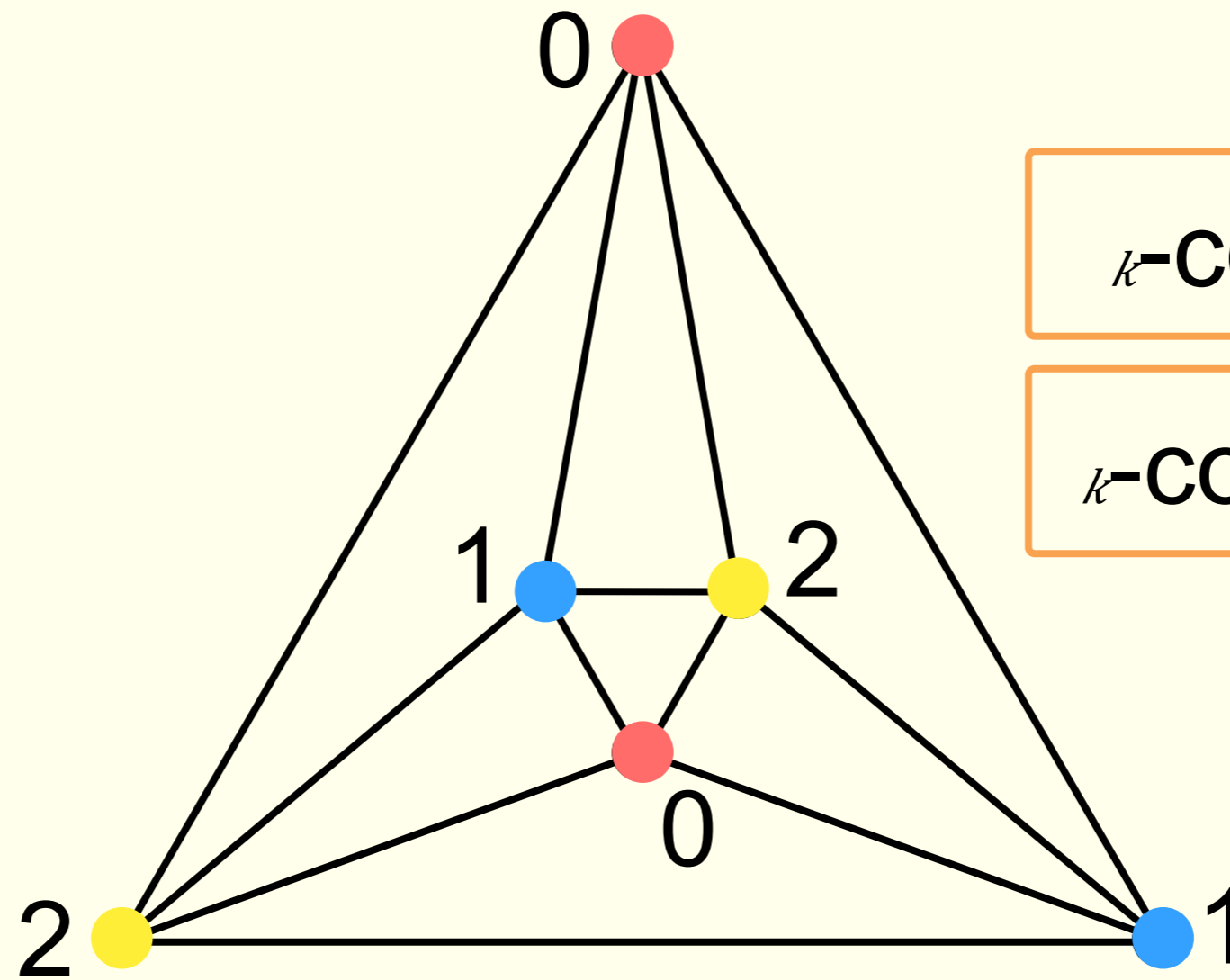
Mi











$k$ -coloring

$k$ -colorable

$\chi(G)$  : the minimum number of colors needed to assign colors to a graph  $G$

$\chi(G) \geq 3$  ( $G$  : a triangulation)

# $n$ -Triad coloring

---

$G$ : a triangulation on a closed surface

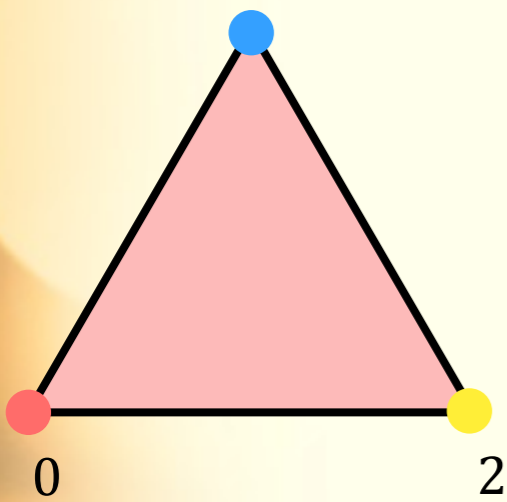
$$\mathcal{T} = \{ \{i, i+1, i+2\} \mid i \in \mathbb{Z} \downarrow n \}$$

$c : V(G) \rightarrow \{1, \dots, n\}$  is called an  **$n$ -triad coloring** if  $\{c(u), c(v), c(w)\}$  belongs to  $\mathcal{T}$  for each face  $uvw$  of  $G$ .

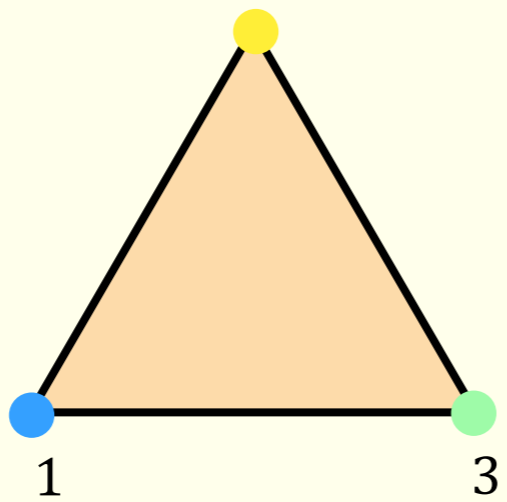


$n=7$

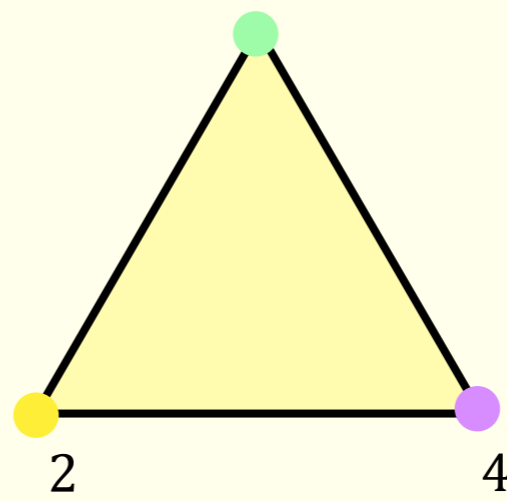
1



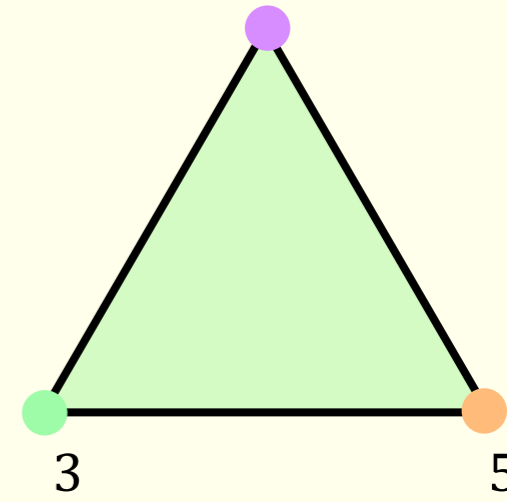
2



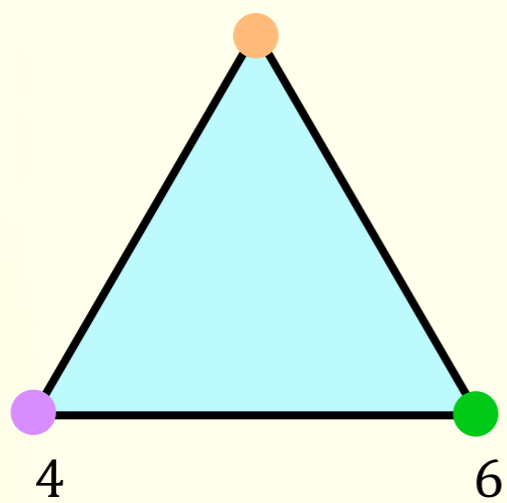
3



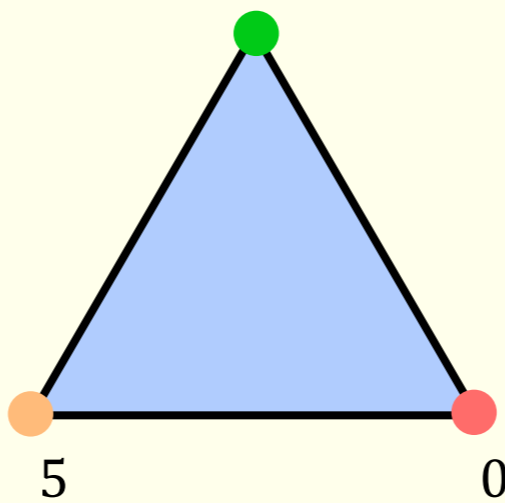
4



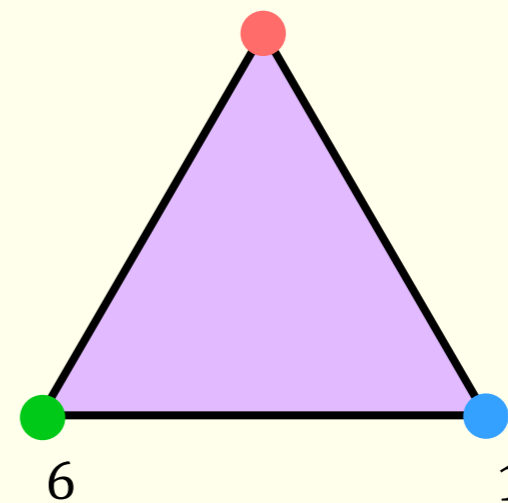
5



6



0



# $n$ -Triad coloring

---

$G$ : a triangulation on a closed surface

$$\mathcal{T} = \{ \{i, i+1, i+2\} \mid i \in \mathbb{Z} \downarrow n \}$$

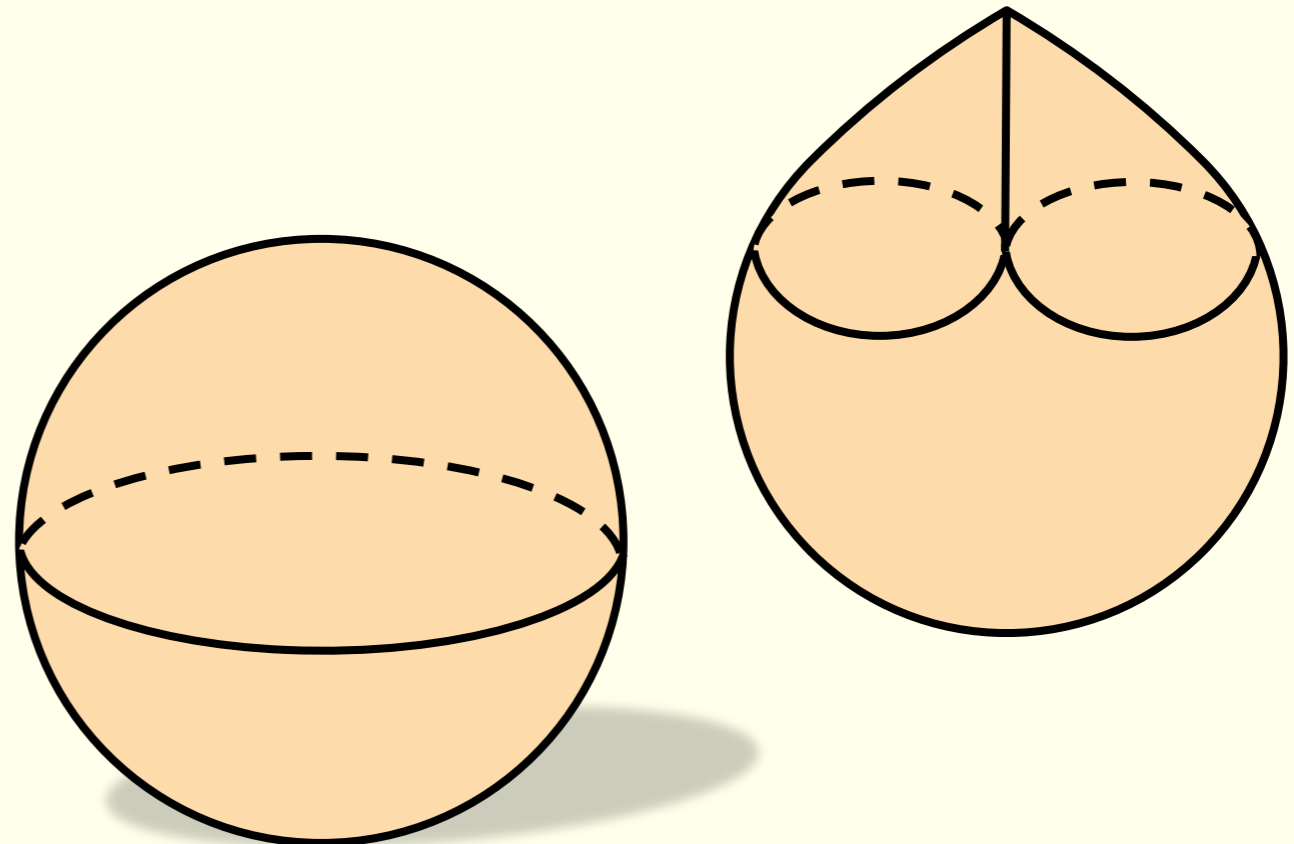
$c : V(G) \rightarrow \{1, \dots, n\}$  is called an  **$n$ -triad coloring** if  $\{c(u), c(v), c(w)\}$  belongs to  $\mathcal{T}$  for each face  $uvw$  of  $G$ .

- \*  $n$ -triad colorable  $\Rightarrow$   $n$ -colorable
- \* 3-colorable  $\Rightarrow$   $n$ -triad colorable

# Main theorem

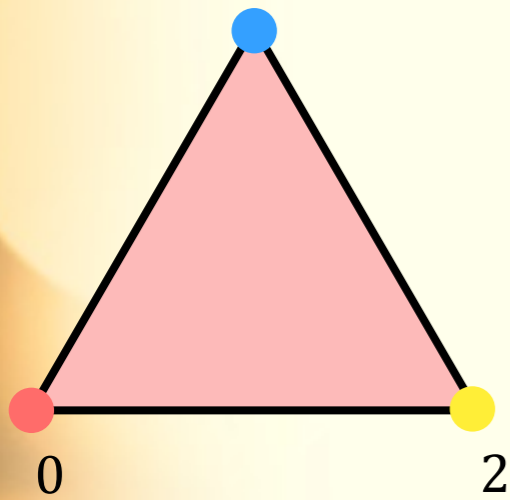
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A triangulation  $G$  on the sphere or the projective plane is  $n$ -triad colorable for some  $n \geq 5$  if and only if  $\chi(G) = 3$ .

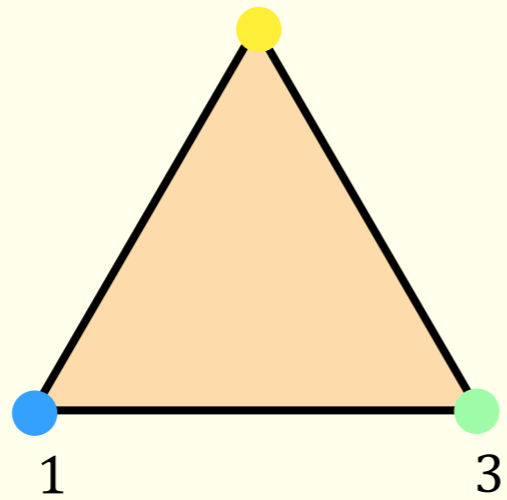


$n=7$

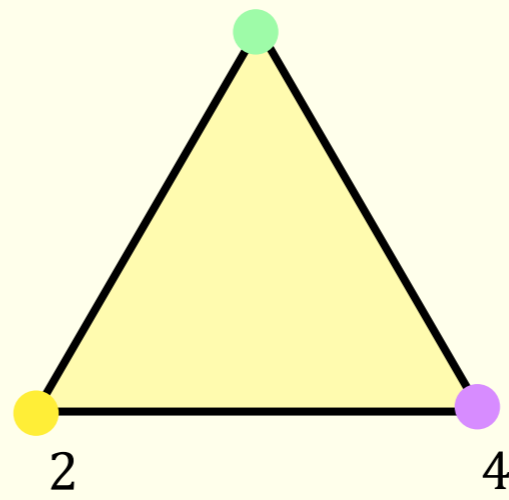
1



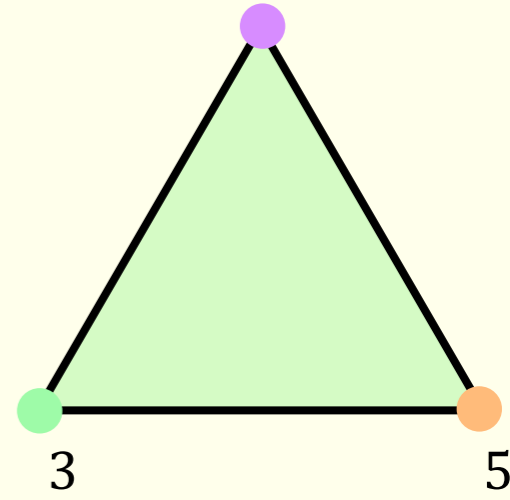
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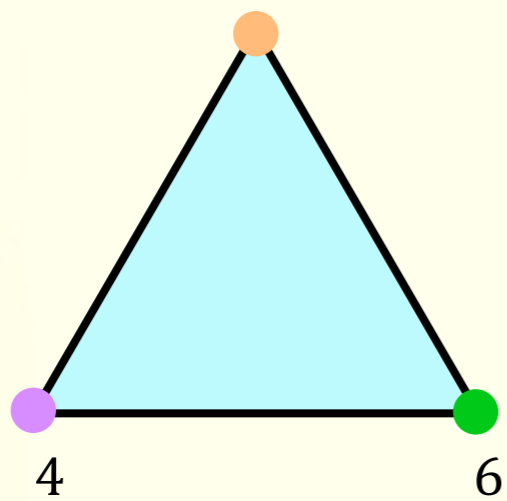
3



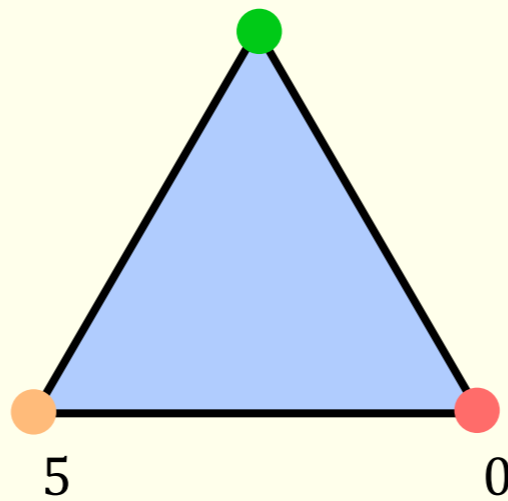
4



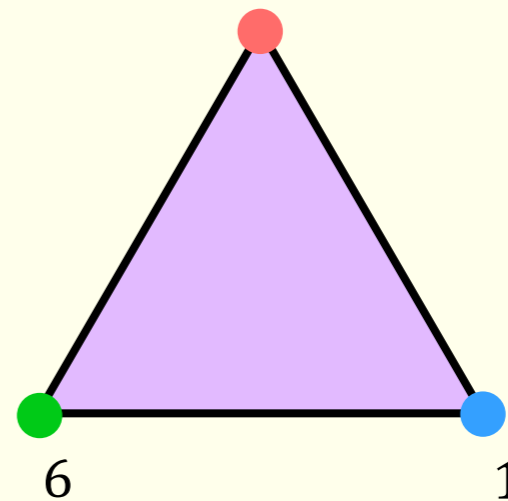
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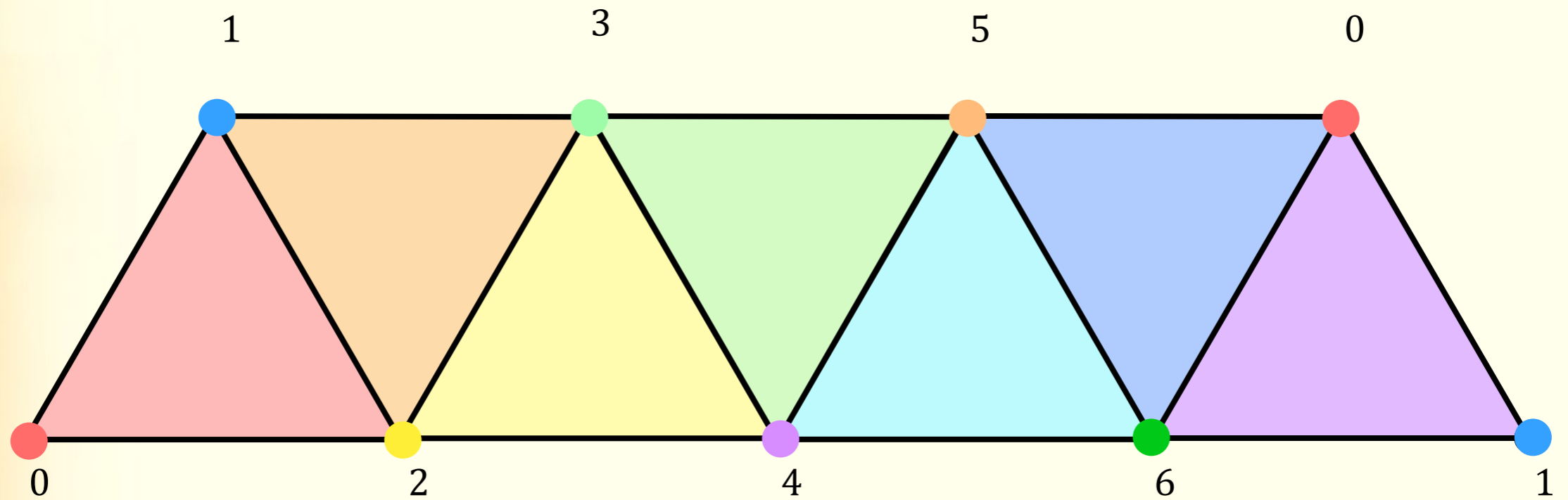
6



0

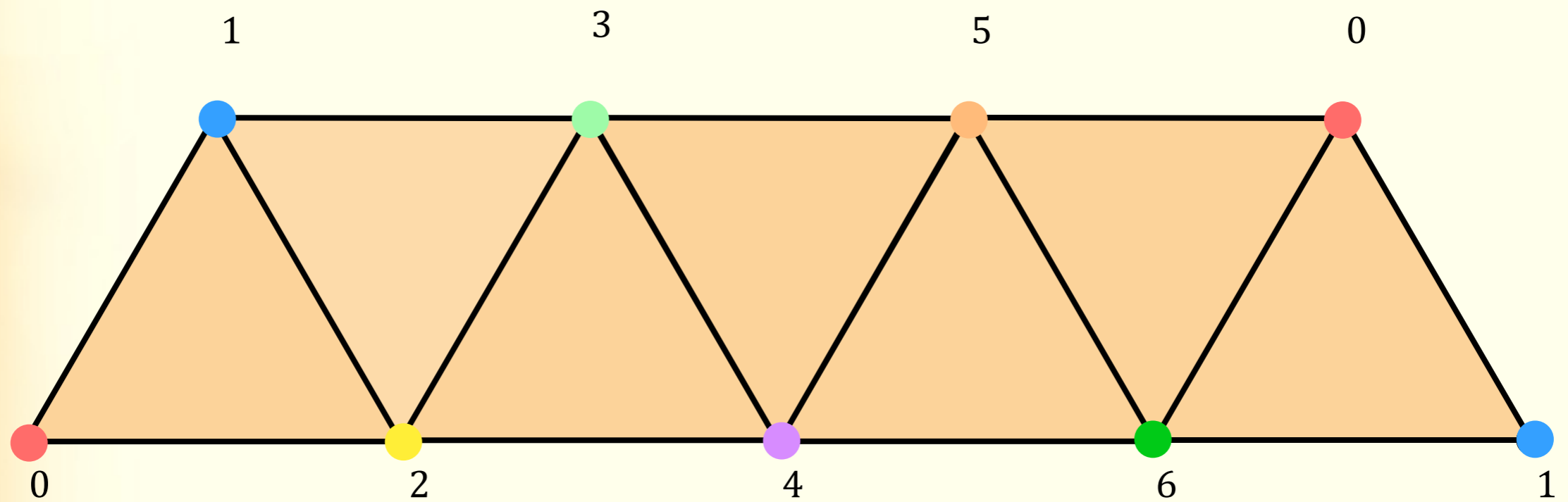


$n=7$



triad complex  $K(\mathcal{T})$

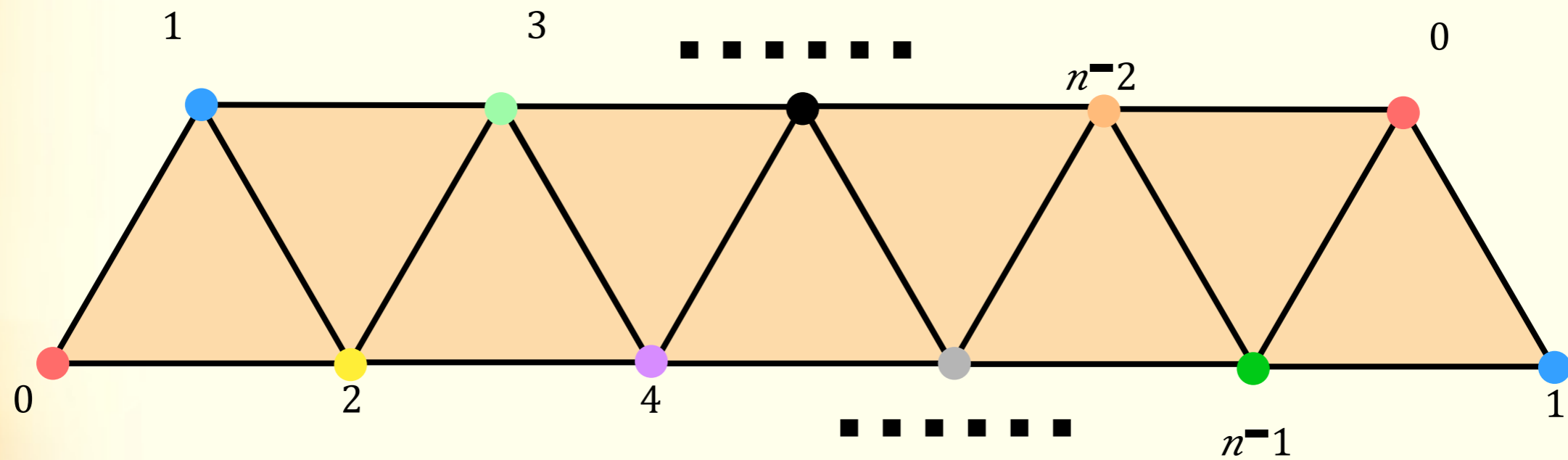
$n=7$



triad space  $X(\mathcal{T})$

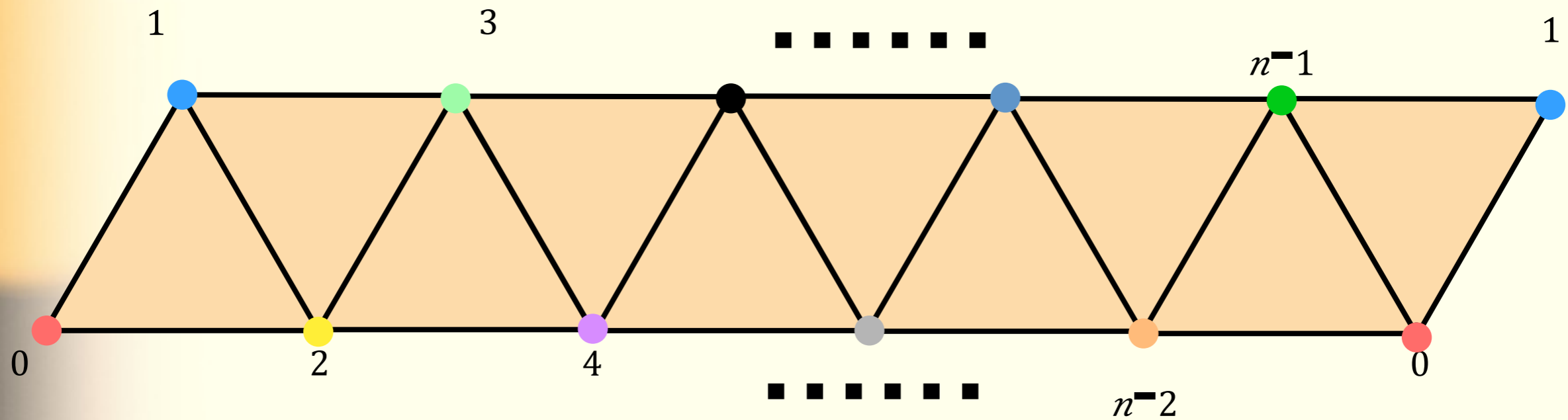
$n \geq 5$

$n$  : odd

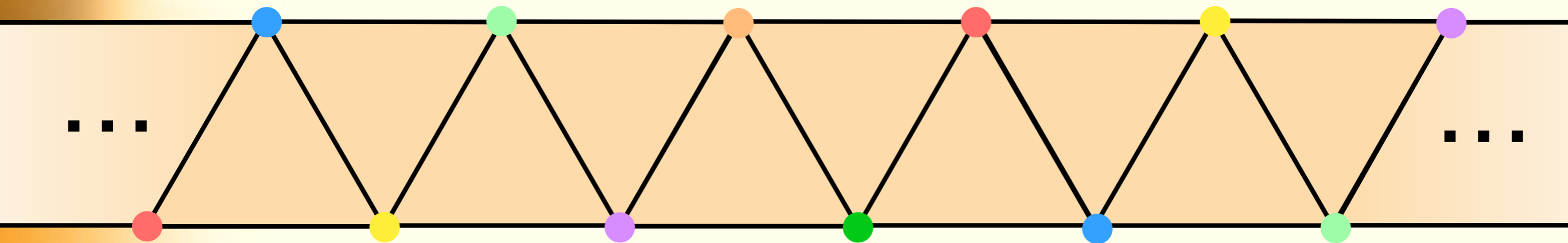


$n \geq 6$

$n$  : even

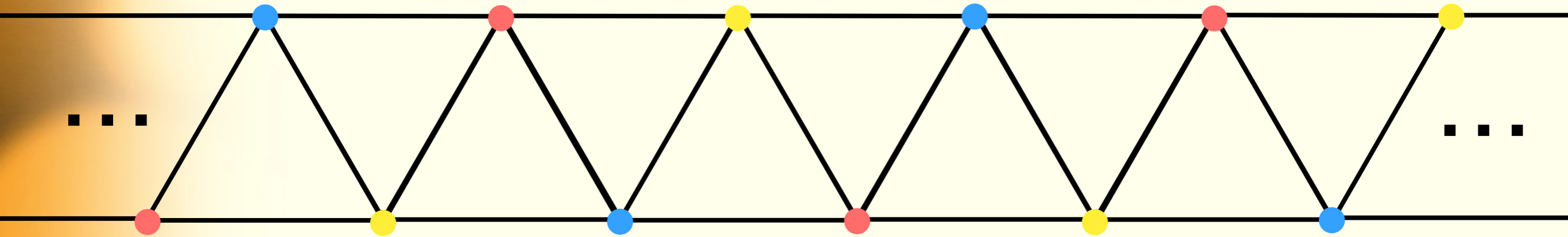


$X(\mathcal{J})$



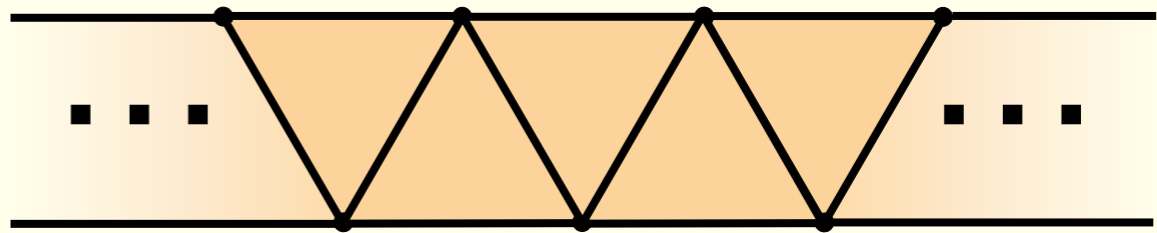


$G(\mathcal{T})$

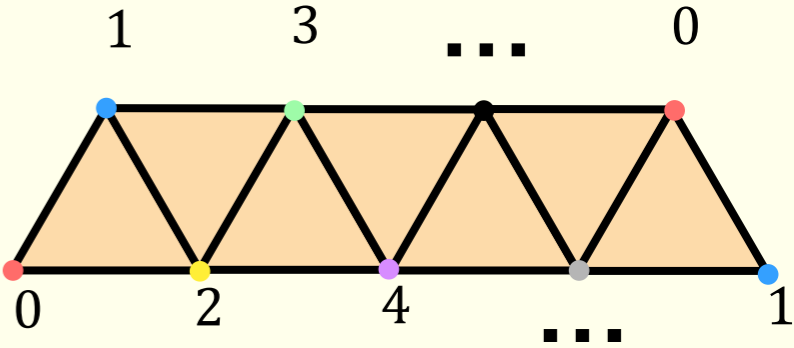
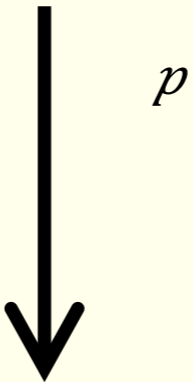


$$\chi(G(\mathcal{T}))=3$$

$$f_{\downarrow c} \downarrow \# (\pi_{\downarrow 1} (F \uparrow 2)) = \{1\}$$



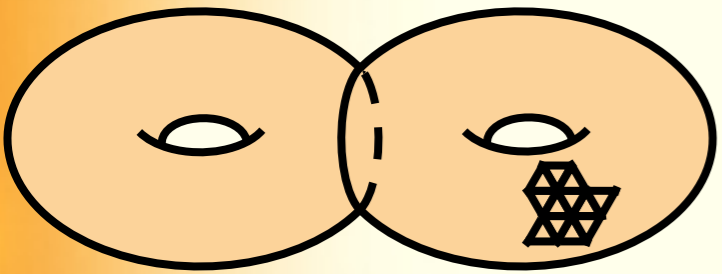
$X(T)$



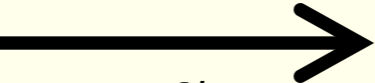
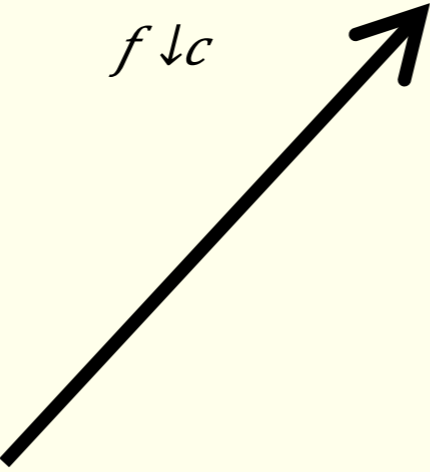
$X(T)$

$\pi_{\downarrow 1} (X(T))$

$\cong \mathbb{Z}$



$F \uparrow 2$



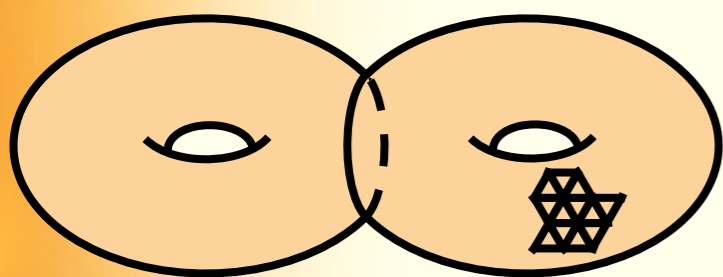
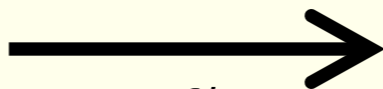
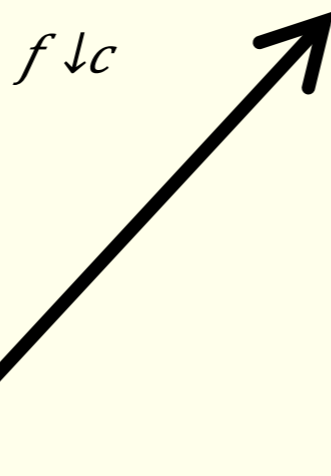
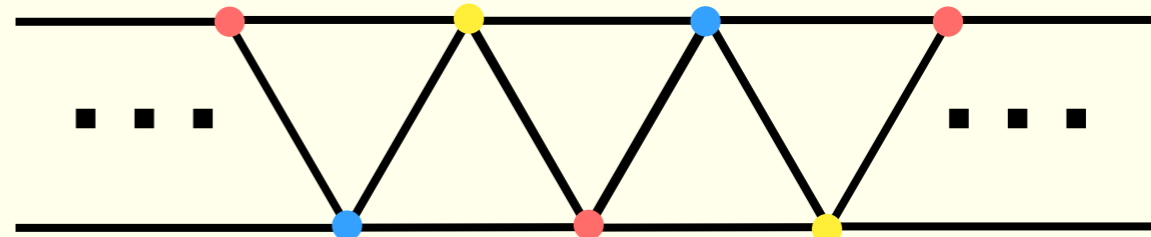
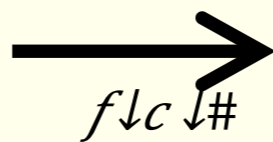
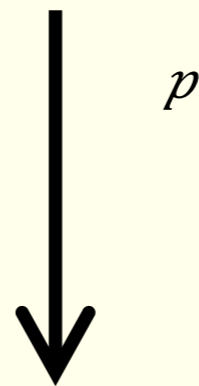
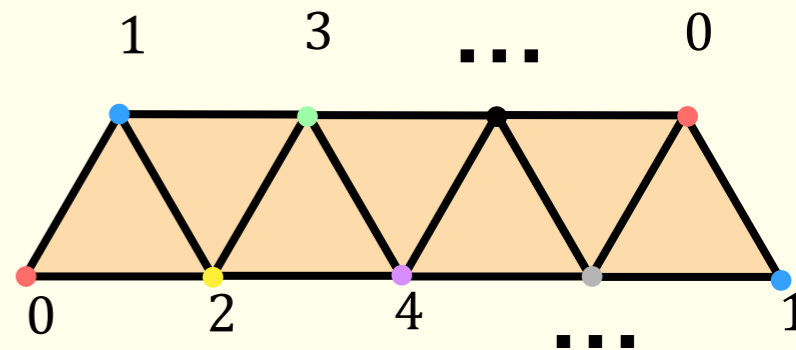
$\pi_{\downarrow 1} (F \uparrow 2)$



$f_{\downarrow c} \downarrow \#$

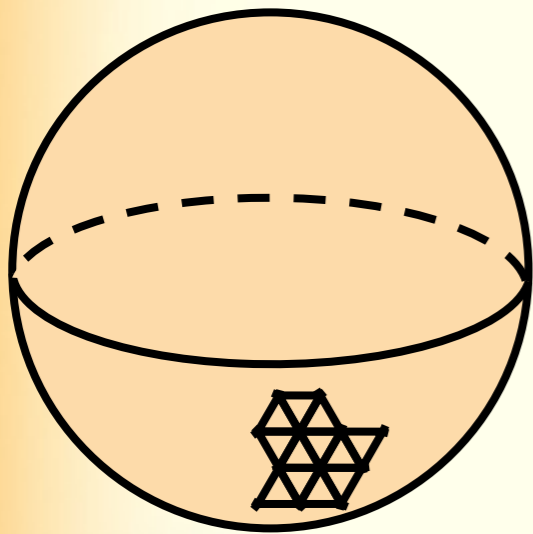
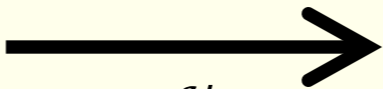
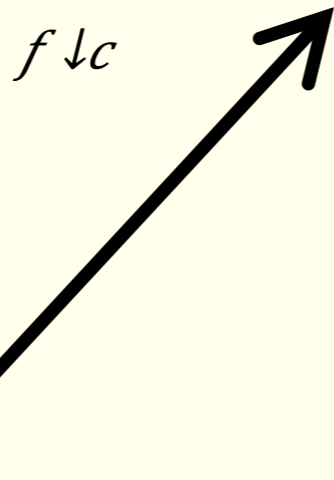
$$f \downarrow c \downarrow \# (\pi \downarrow 1 (F \uparrow 2)) = \{1\}$$

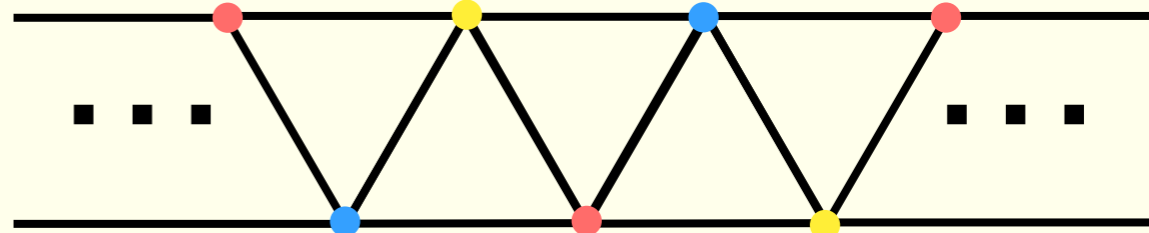
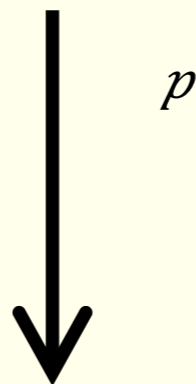
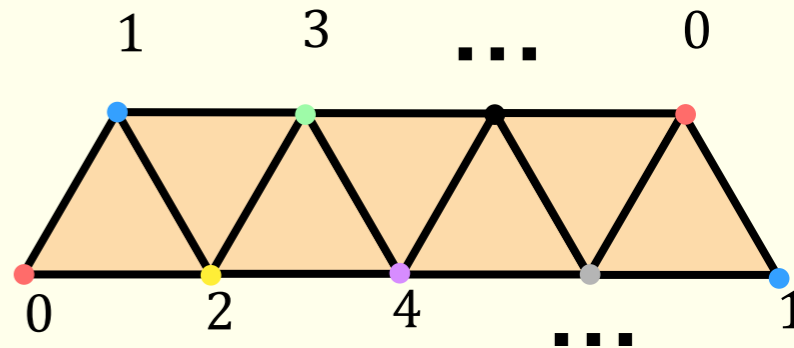
$$\chi(G) = 3$$


 $F \uparrow 2$ 

 $\pi \downarrow 1 (F \uparrow 2)$ 

 $X(\mathcal{T})$ 
 $G(\mathcal{T})$ 

 $X(\mathcal{T})$ 

 $\pi \downarrow 1 (X(\mathcal{T}))$ 
 $\cong \mathbb{Z}$

$$f \downarrow c \downarrow \# (\pi \downarrow 1 (S \uparrow 2)) = \{1\}$$

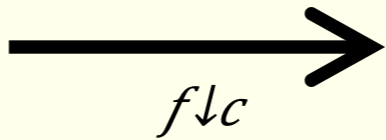
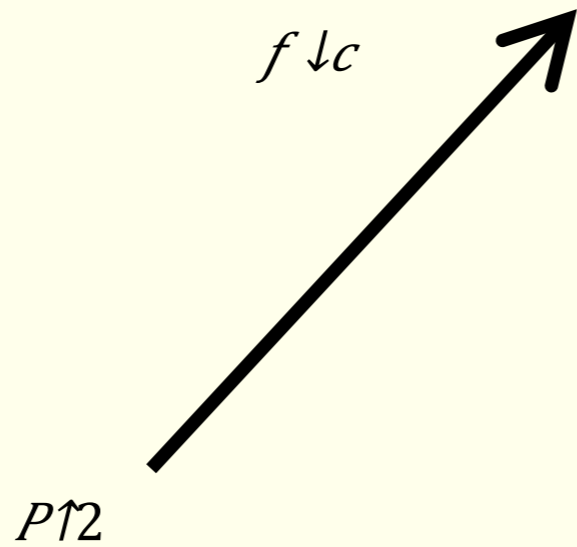
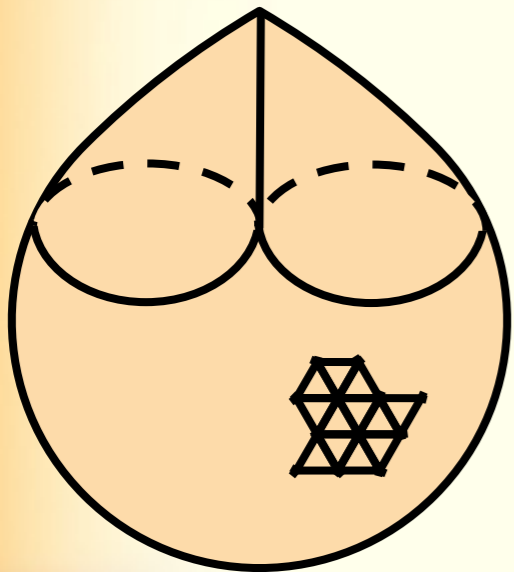
$$\chi(G) = 3$$


 $S \uparrow 2$ 

 $\pi \downarrow 1 (S \uparrow 2)$ 

 $\cong \{1\}$ 

 $X(\mathcal{T})$ 
 $G(\mathcal{T})$ 

 $X(\mathcal{T})$ 

 $\pi \downarrow 1 (X(\mathcal{T}))$ 
 $\cong \mathbb{Z}$

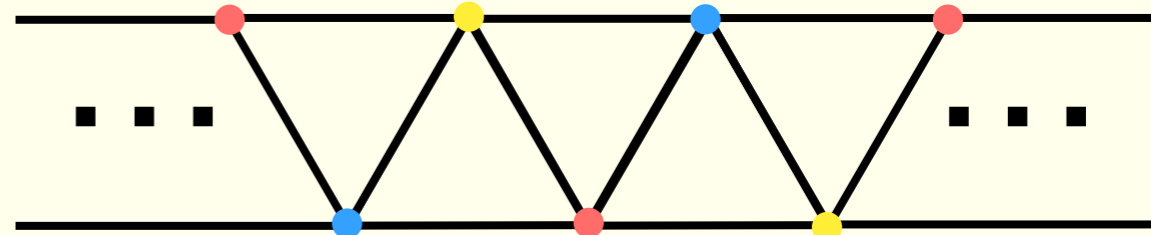
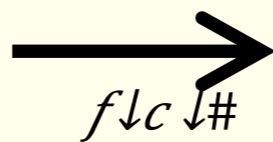
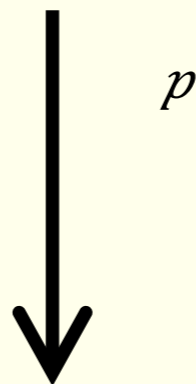
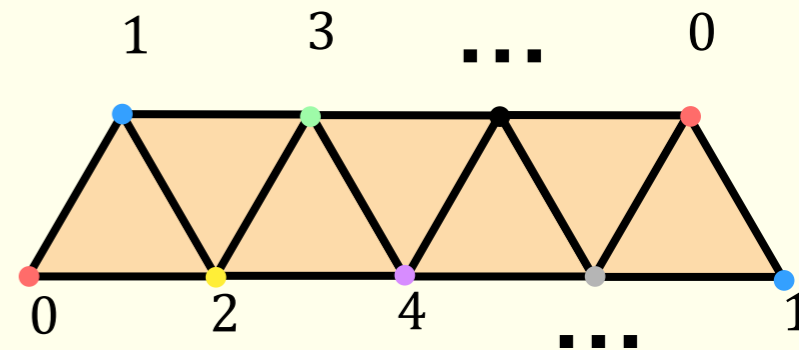
$$f \downarrow c \downarrow \# (\pi \downarrow 1 (P \uparrow 2)) = \{1\}$$

$$\chi(G) = 3$$



$$\pi \downarrow 1 (P \uparrow 2)$$

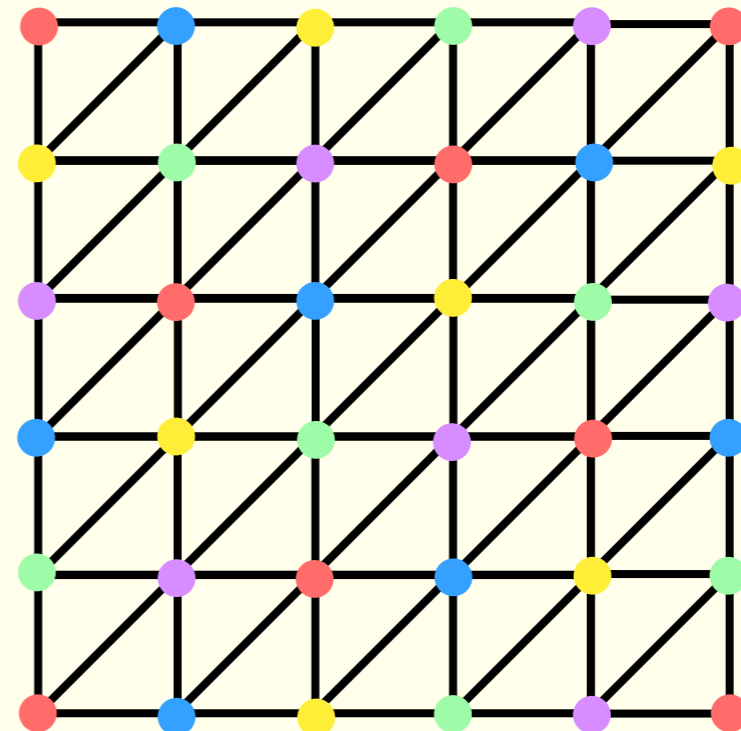
$$\cong \mathbb{Z} \downarrow 2$$


 $X(\mathcal{T})$ 
 $G(\mathcal{T})$ 

 $X(\mathcal{T})$ 

 $\pi \downarrow 1 (X(\mathcal{T}))$ 

$$\cong \mathbb{Z}$$

# Main theorem

A triangulation  $G$  on the sphere or the projective plane is  $n$ -triad colorable for some  $n \geq 5$  if and only if  $\chi(G) = 3$ .

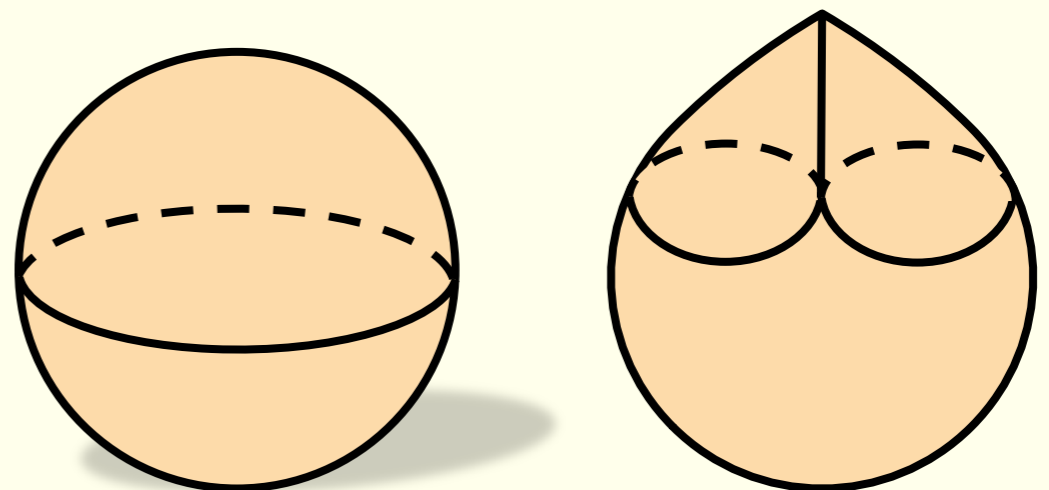


5-triad colorable,  $\chi(G) = 4$

# Remarks

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- \* Any triangulation on the sphere is 4-triad colorable.
- \* There are many triangulations on the projective plane which are *not*  $n$ -triad colorable for any  $n \geq 3$ .



*Thank you for your attention!!*

