Triad colorings of triangulations on closed surfaces

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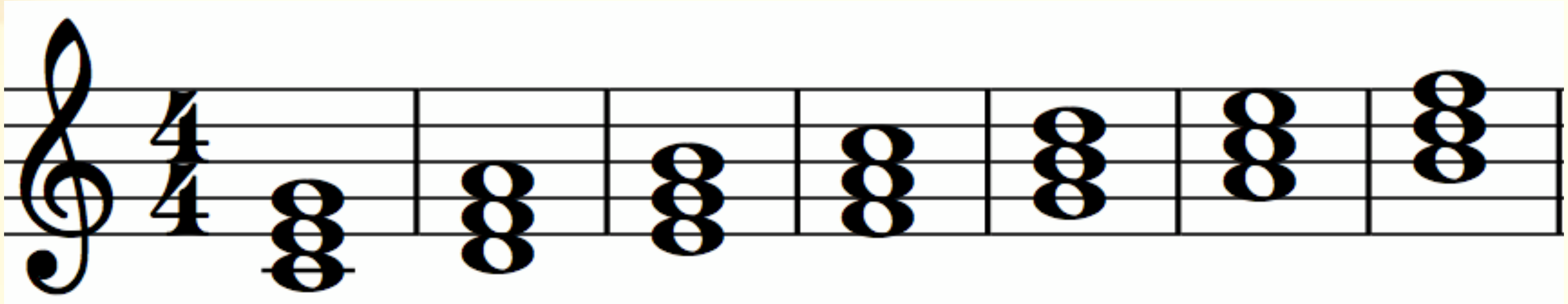
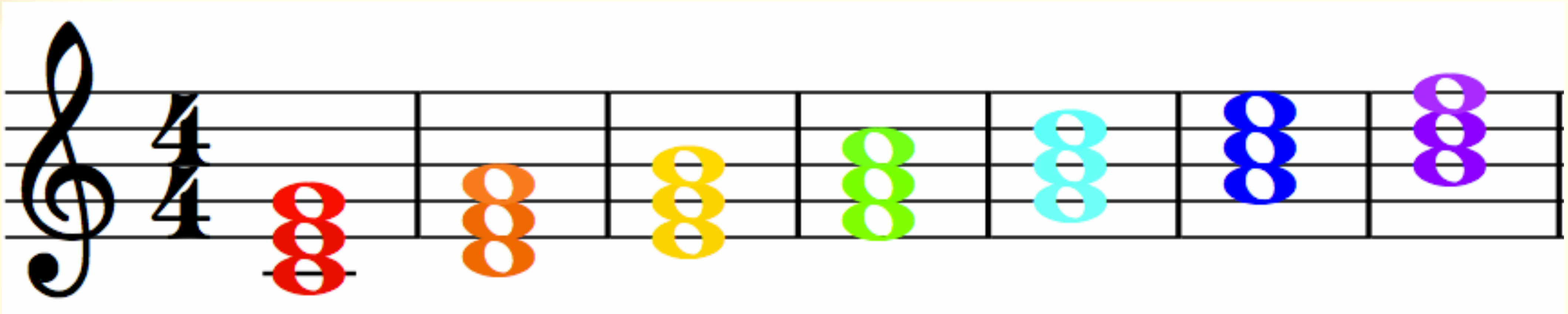
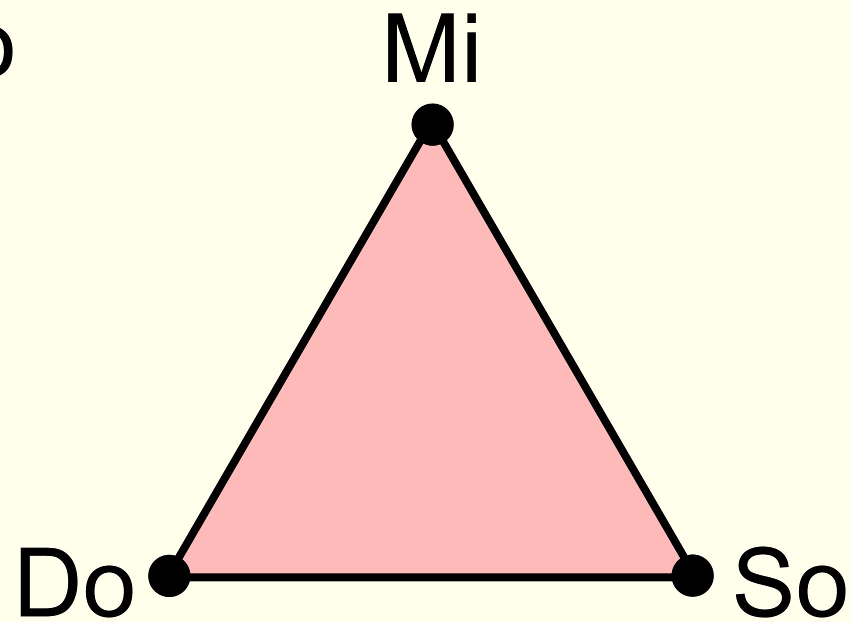
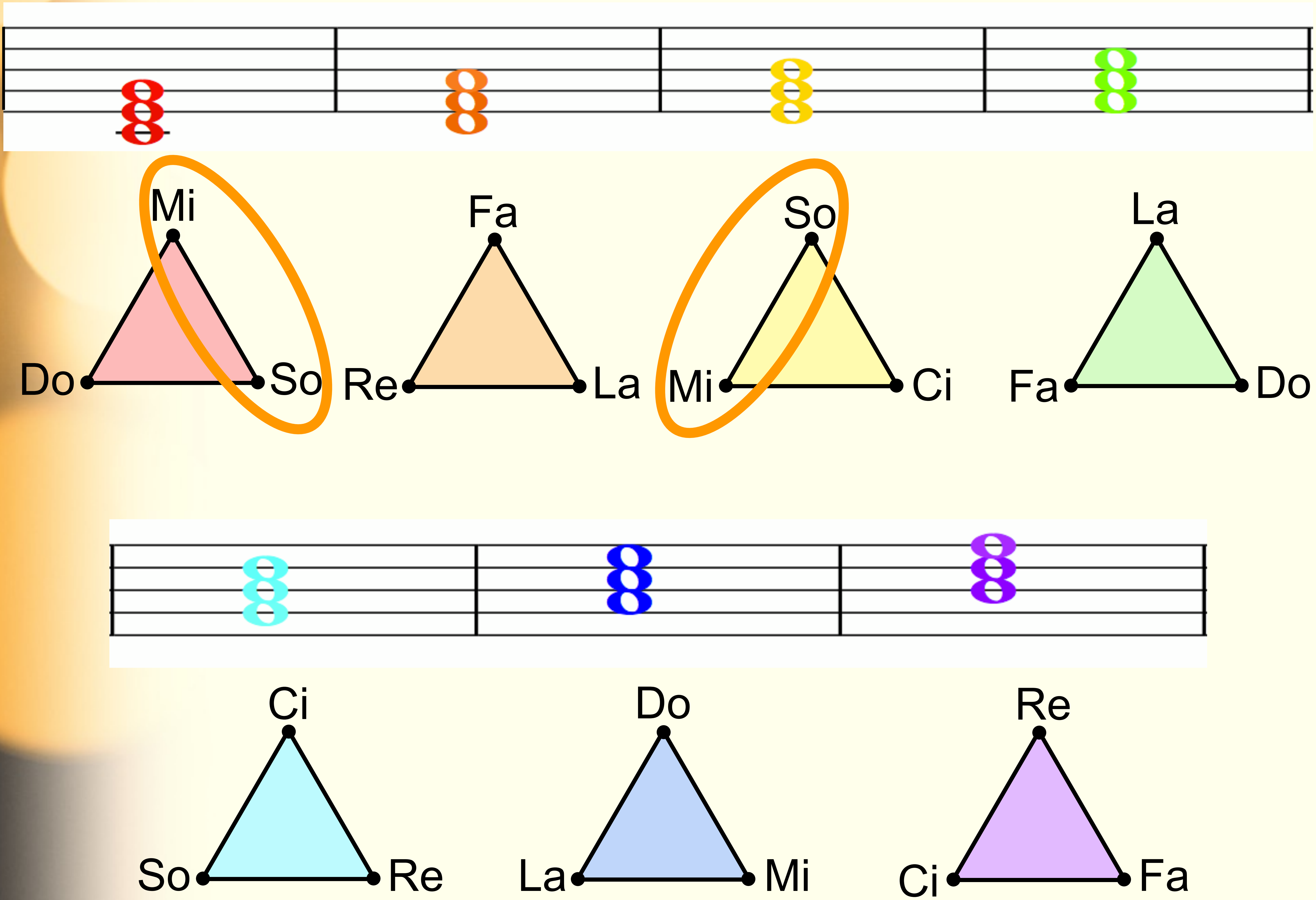
Yokohama National University M1





Do Mi So

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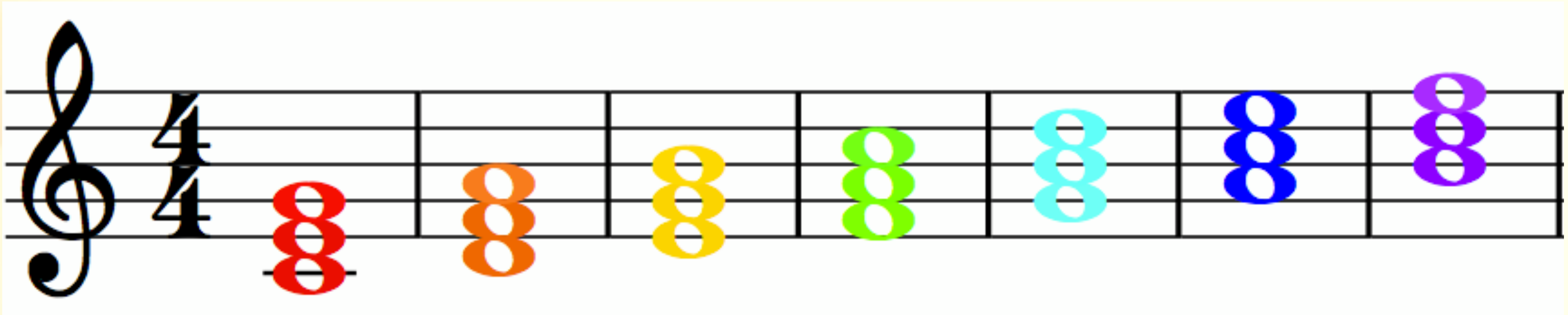

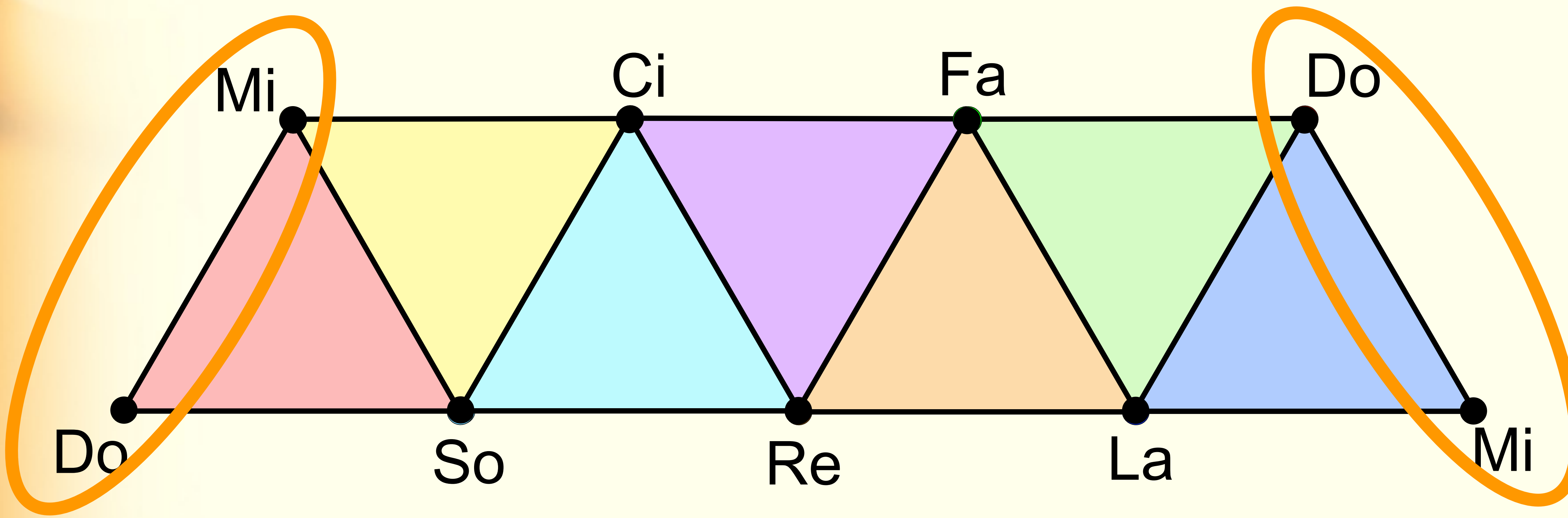

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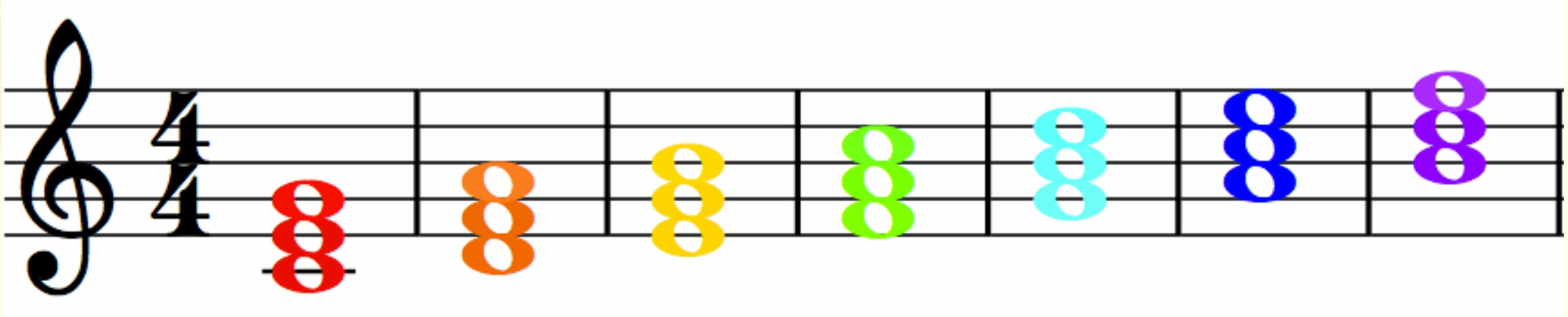

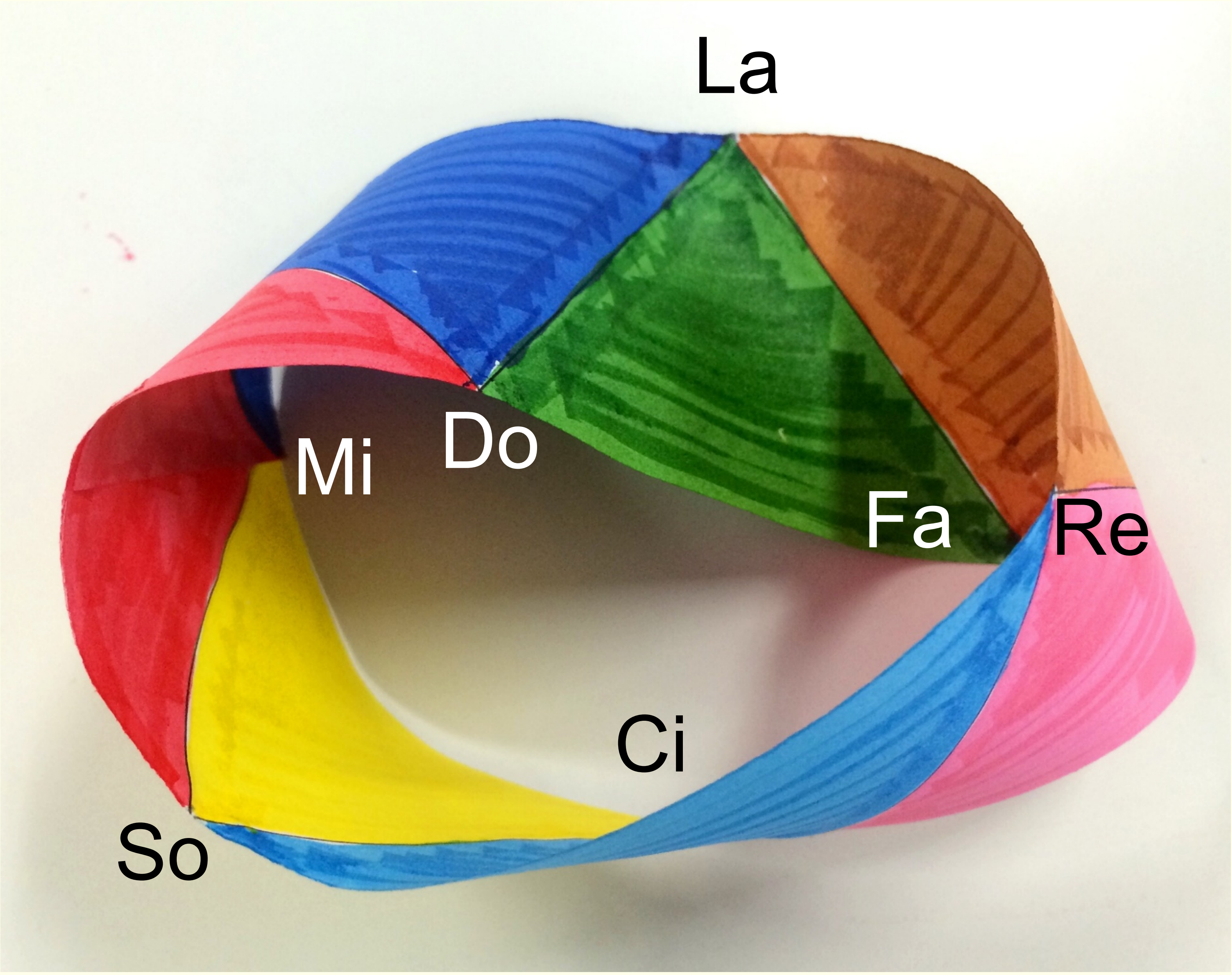
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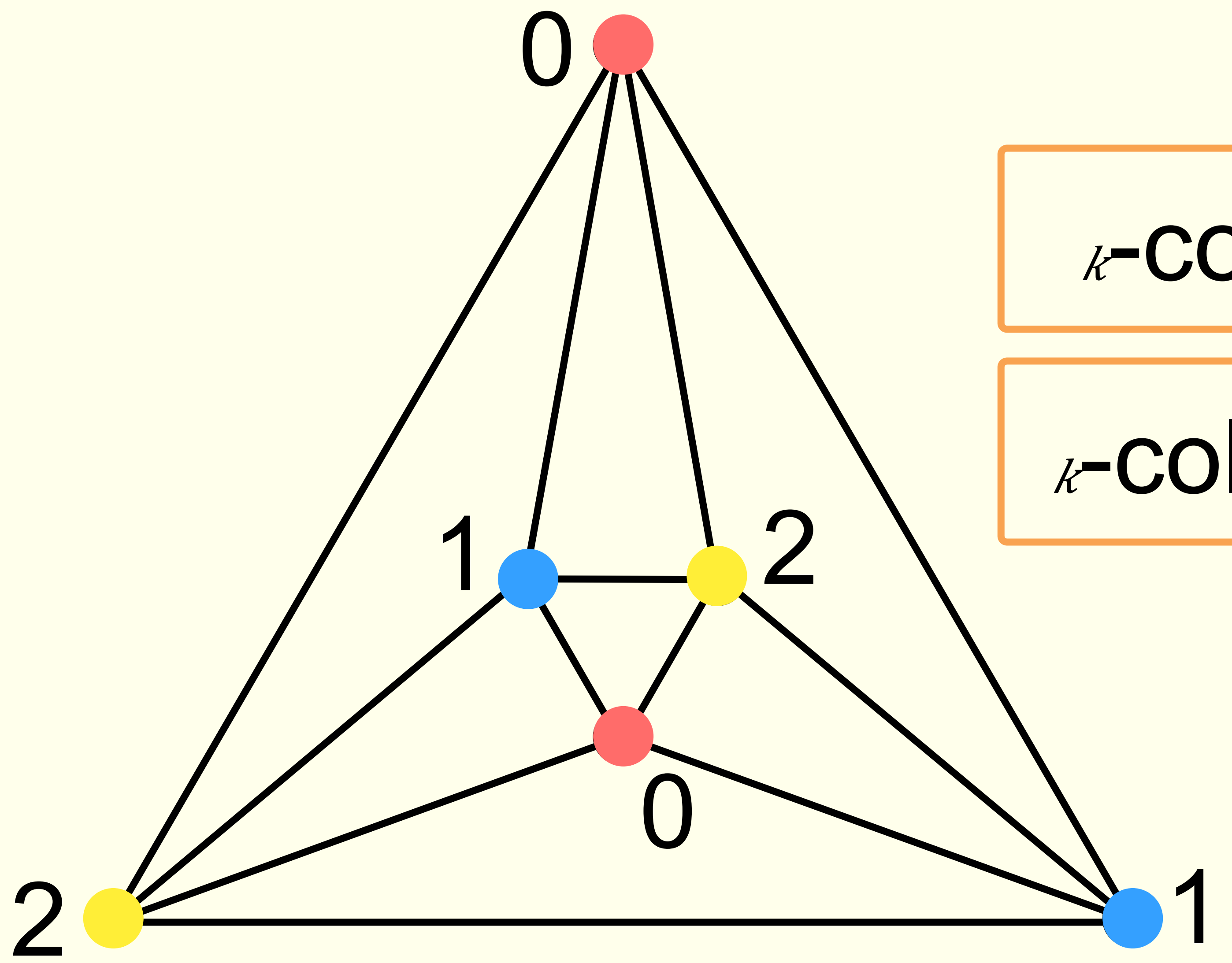


k-coloring

k-colorable

$\chi(G)$: the minimum number of colors needed to assign colors to a graph G

$\chi(G) \geqq 3$ (G : a triangulation)

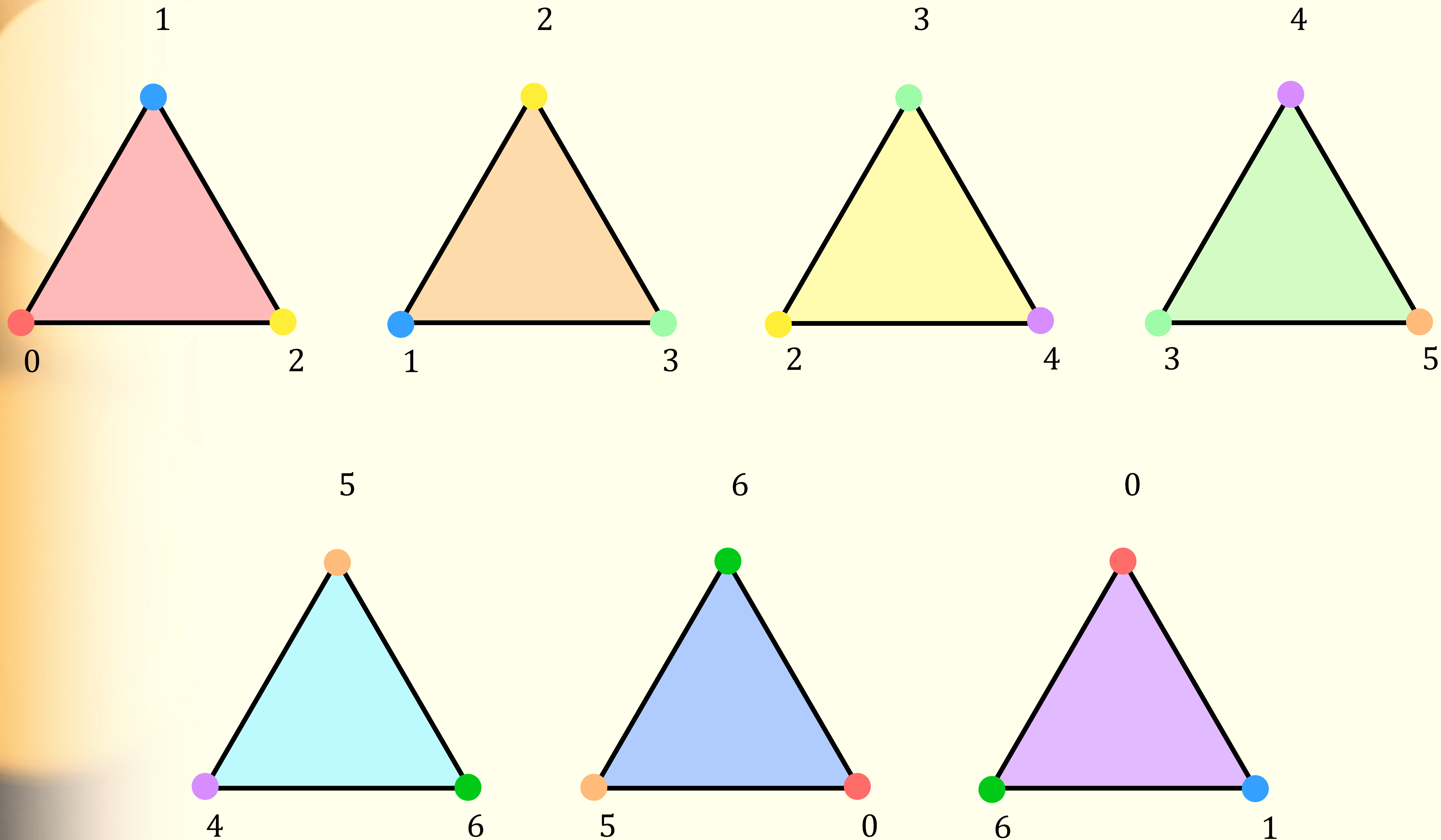
n-Triad coloring

G : a triangulation on a closed surface

$\mathcal{T}=\{\{i,i+1,i+2\} \mid i\in\mathbb{Z}_n\}$

$c : V(G)\to\{1,\dots,n\}$ is called an ***n*-triad coloring** if $\{c(u),c(v),c(w)\}$ belongs to $\mathcal{T}$ for each face uvw of G.

$n=7$



n-Triad coloring

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- n-triad colorable ⇒ n-colorable
- 3-colorable ⇒ n-triad colorable

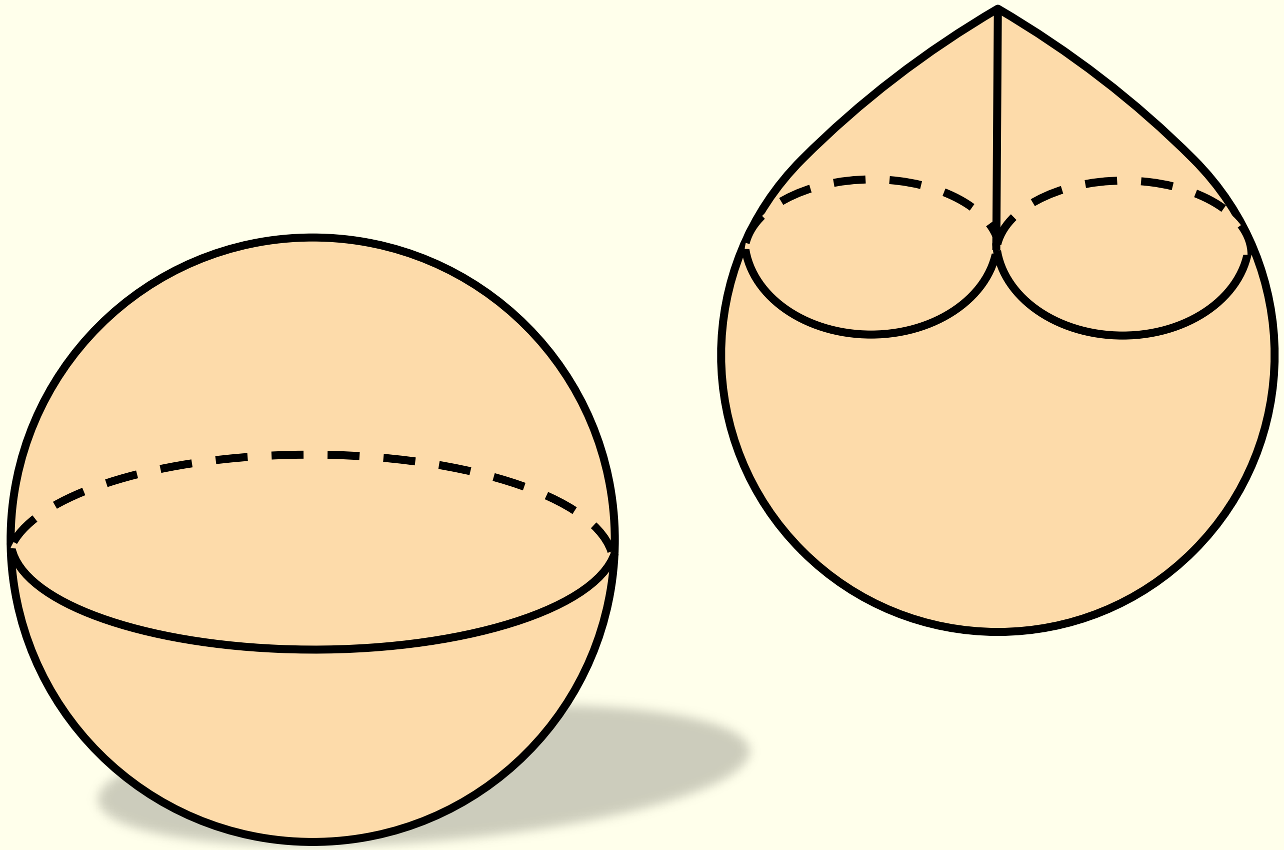
Main theorem

A triangulation G on the sphere or the projective plane is n-triad colorable for some $n \geqq 5$ if and only if $\chi(G)=3$.

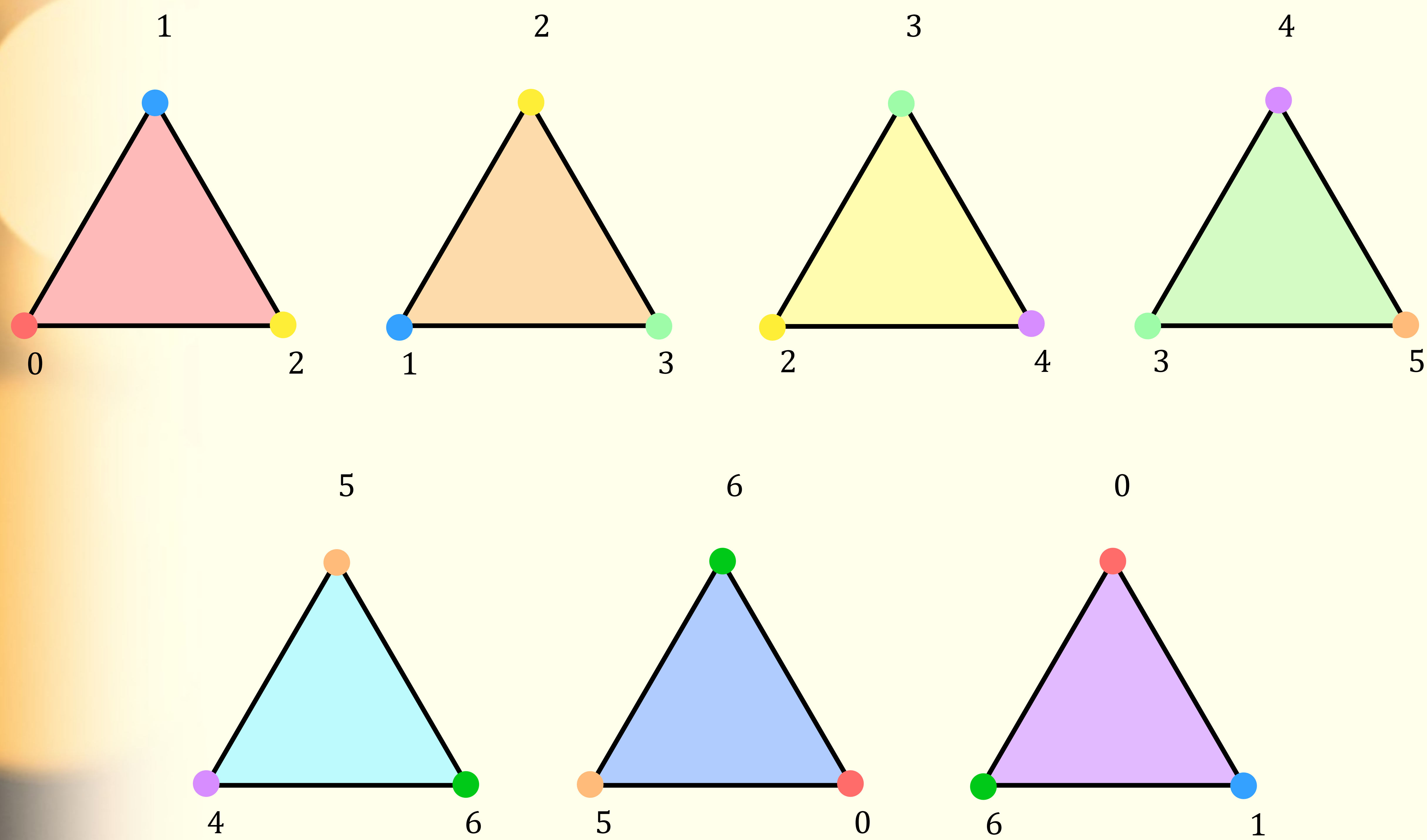
$\Leftarrow$

$\Rightarrow$

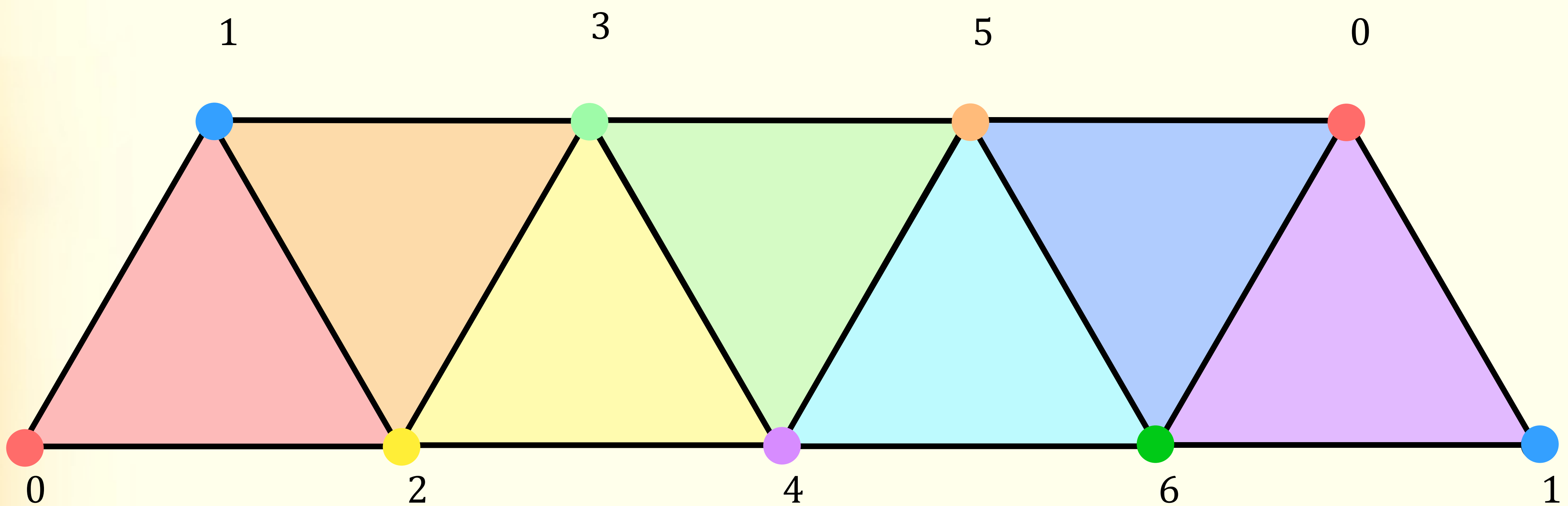
?



$n=7$

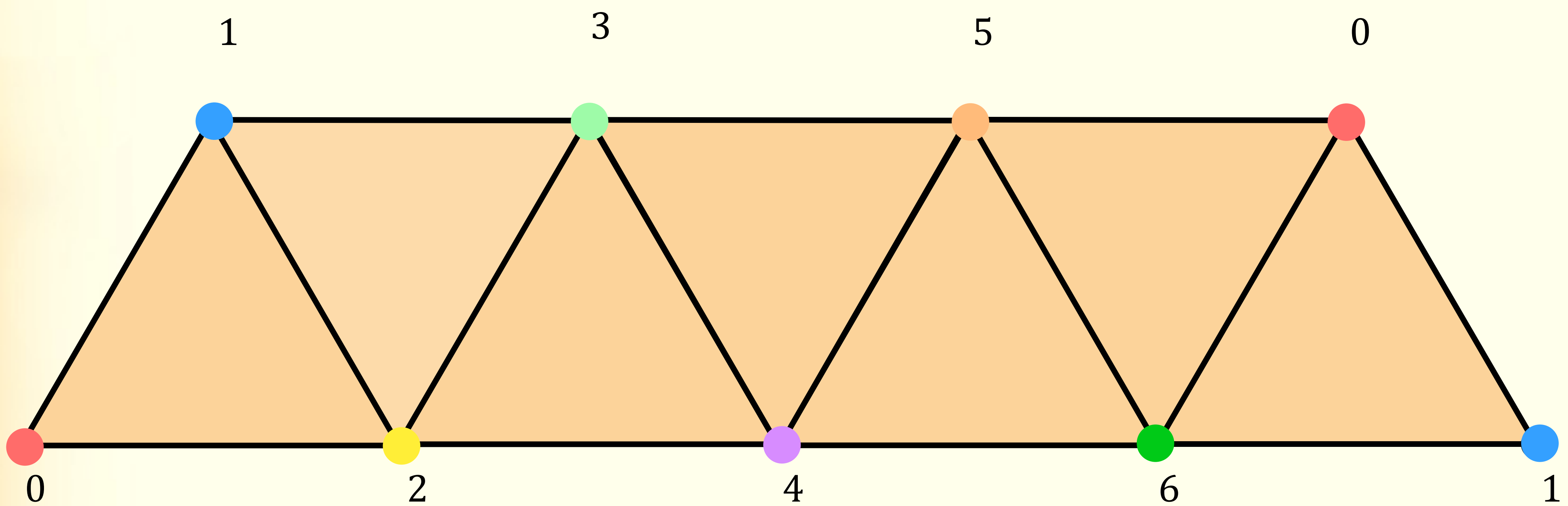


n=7



triad complex $K(\mathcal{T})$

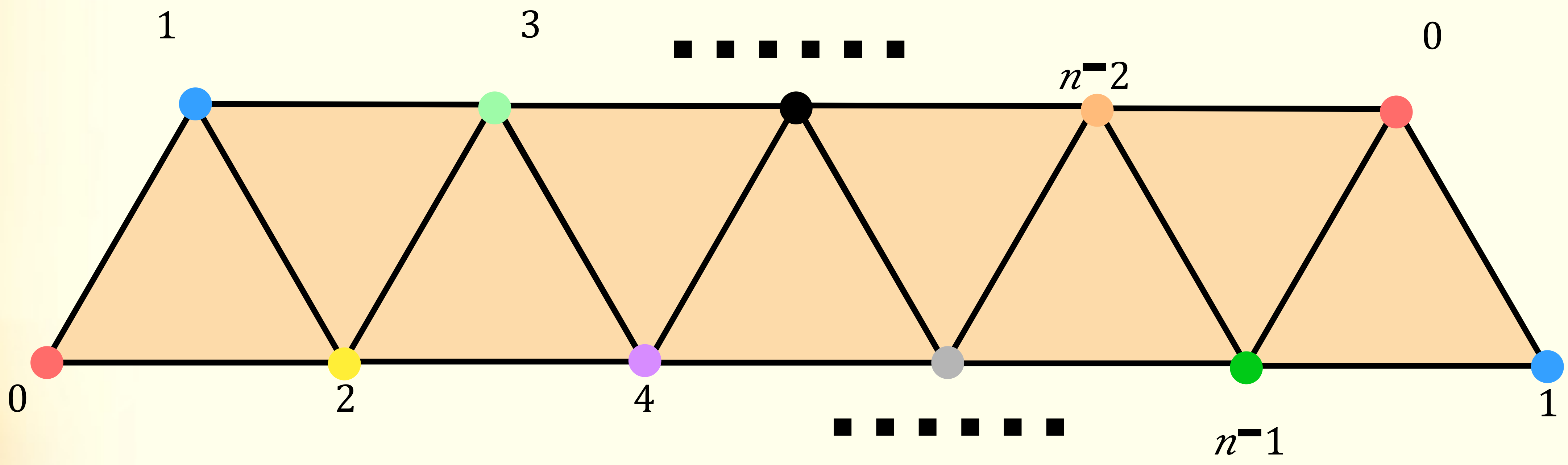
n=7



triad space $X(\mathcal{T})$

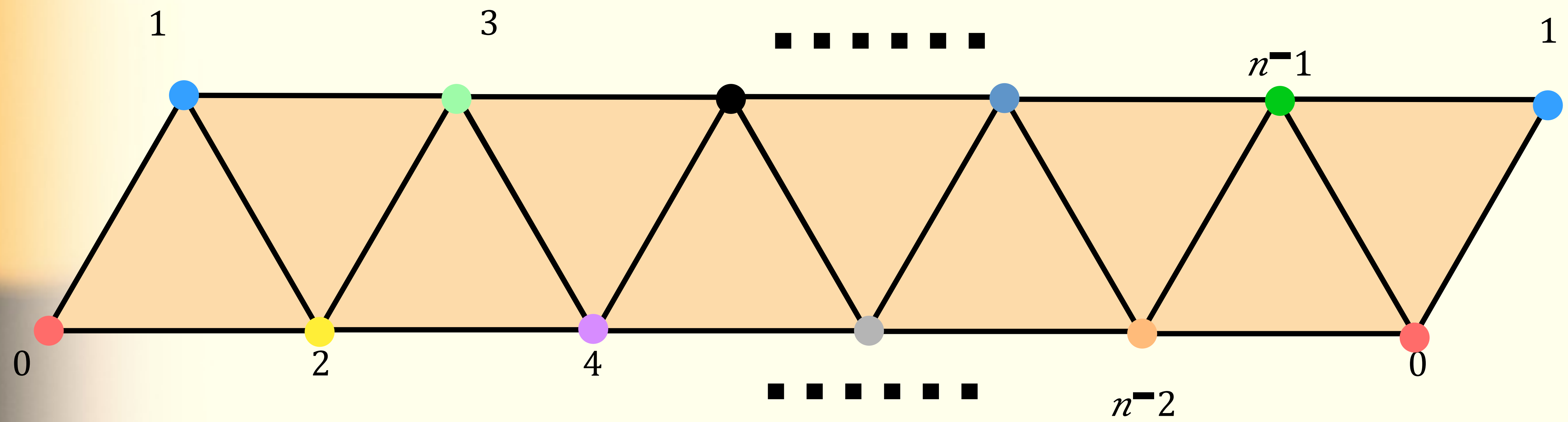
$n \geqq 5$

n : odd

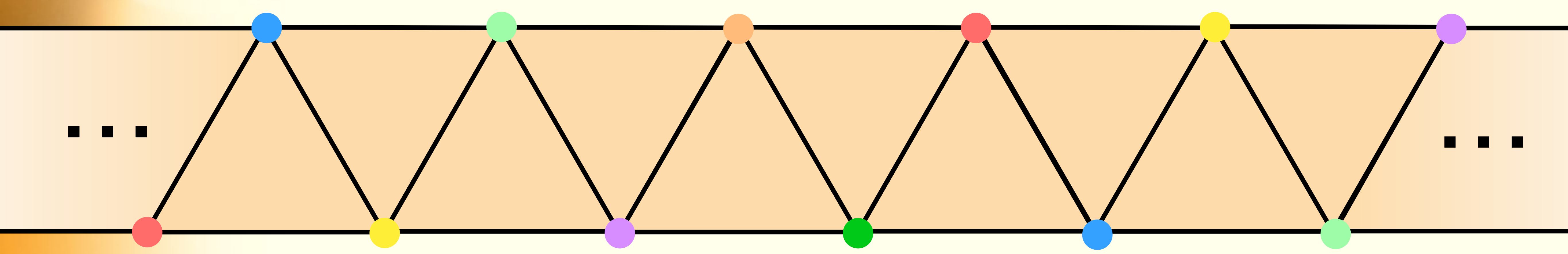


$n \geqq 6$

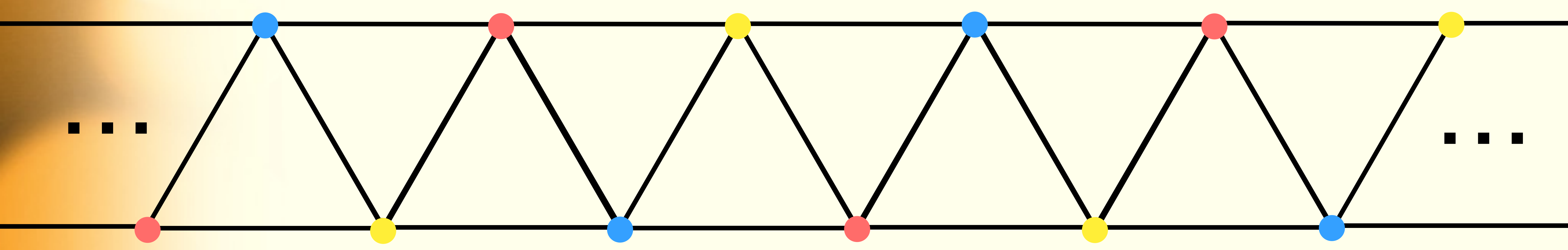
n : even



$X(\mathcal{T})$

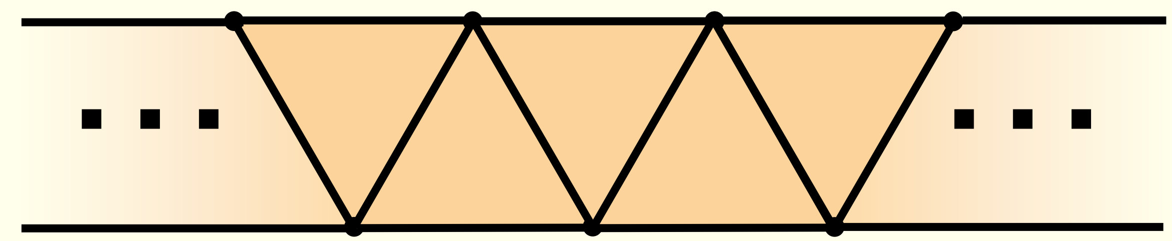


$\mathcal{G}(\mathcal{T})$



$\chi(\mathcal{G}(\mathcal{T}))=3$

$f_{c\#}(\pi_1(F^2))=\{1\}$

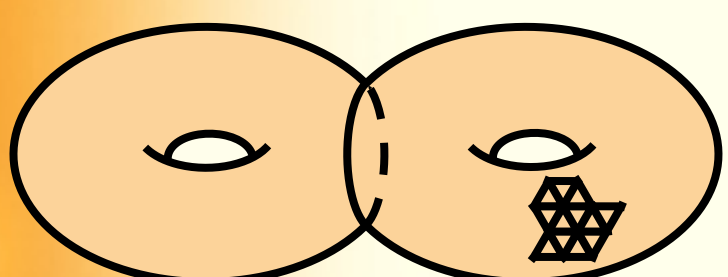


$X(\mathcal{T})$

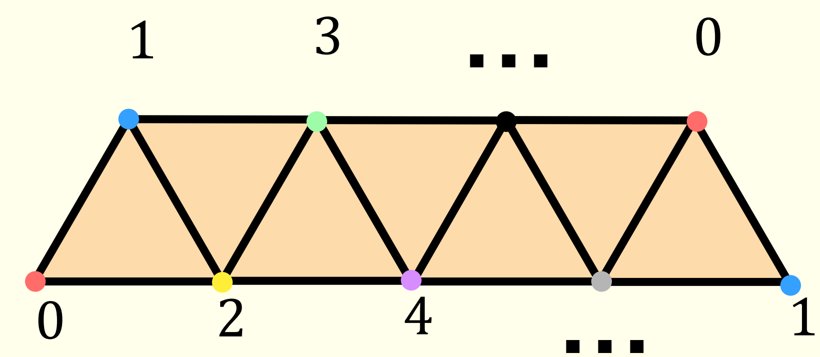
f_c

p

F^2



$X(\mathcal{T})$



f_c

$\pi_1(F^2)$

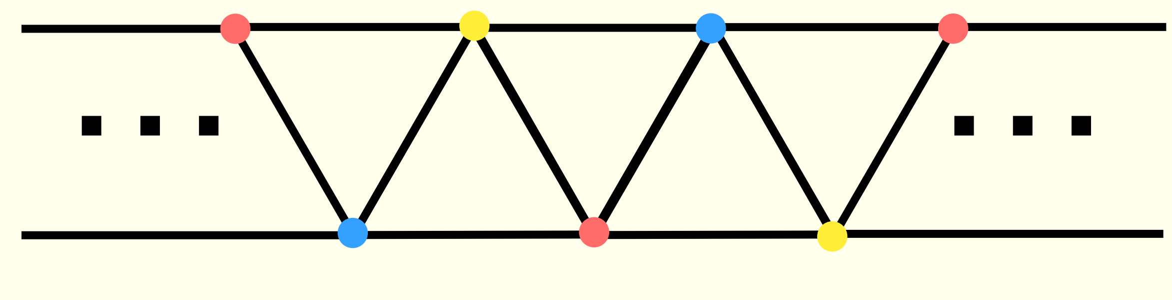
$\pi_1(X(\mathcal{T}))$

$f_{c\#}$

$\cong \mathbb{Z}$

$f_{c\#}(\pi_1(F^2))=\{1\}$

$\chi(G)=3$



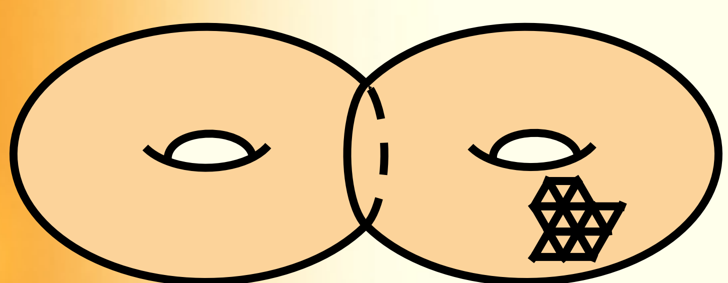
f_c

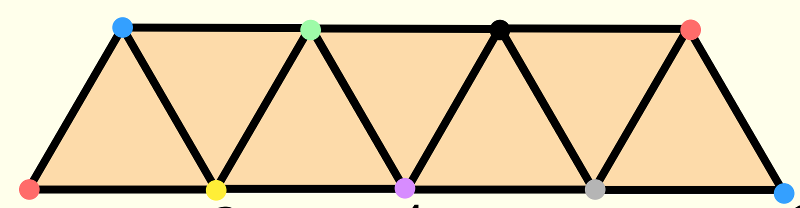
p

F^2

f_c

X$(\mathcal{T})$



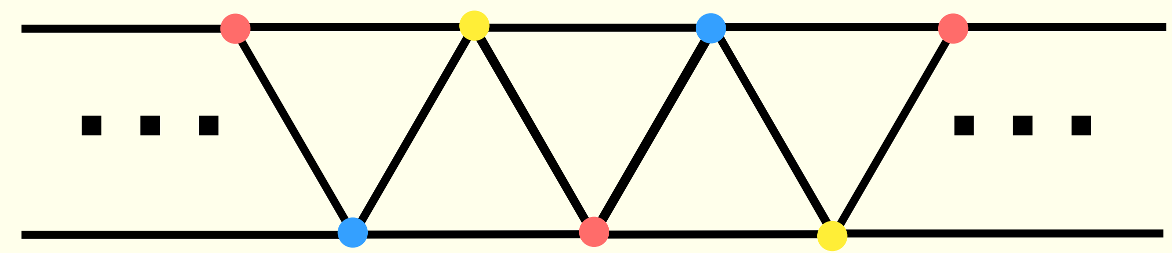


$\pi_1(F^2)$ $\xrightarrow{f_{c\#}}$ $\pi_1(X(\mathcal{T}))$

$\cong\mathbb{Z}$

$f_{c\#}(\pi_1(S^2))=\{1\}$

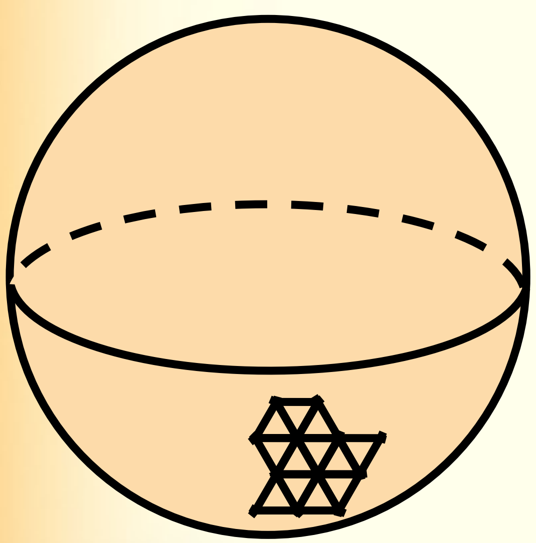
$\chi(G)=3$



$X(\mathcal{T})$ $G(\mathcal{T})$

f_c

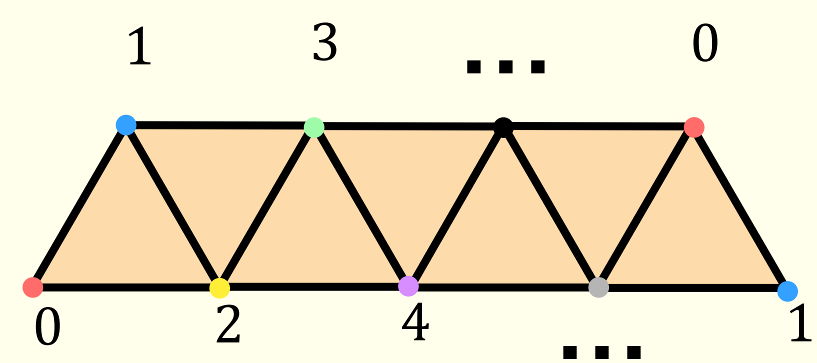
p



S^2

f_c

$X(\mathcal{T})$



$\pi_1(S^2)$

$f_{c\#}$

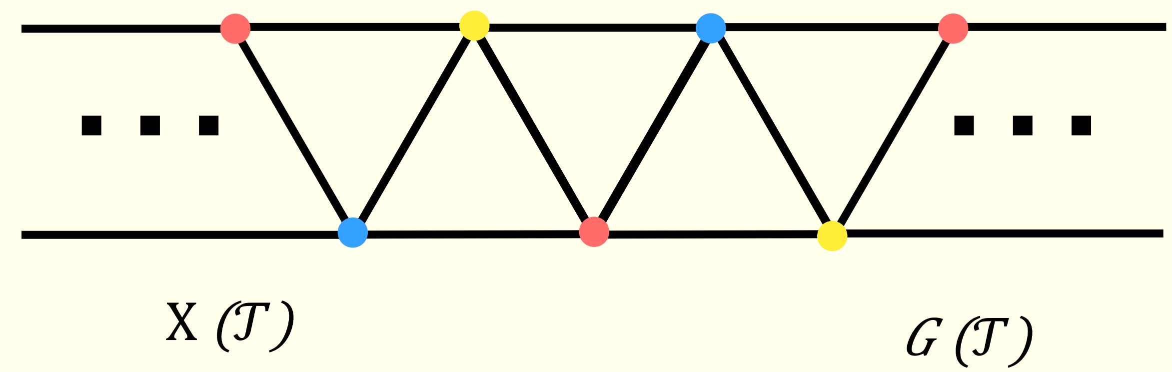
$\pi_1(X(\mathcal{T}))$

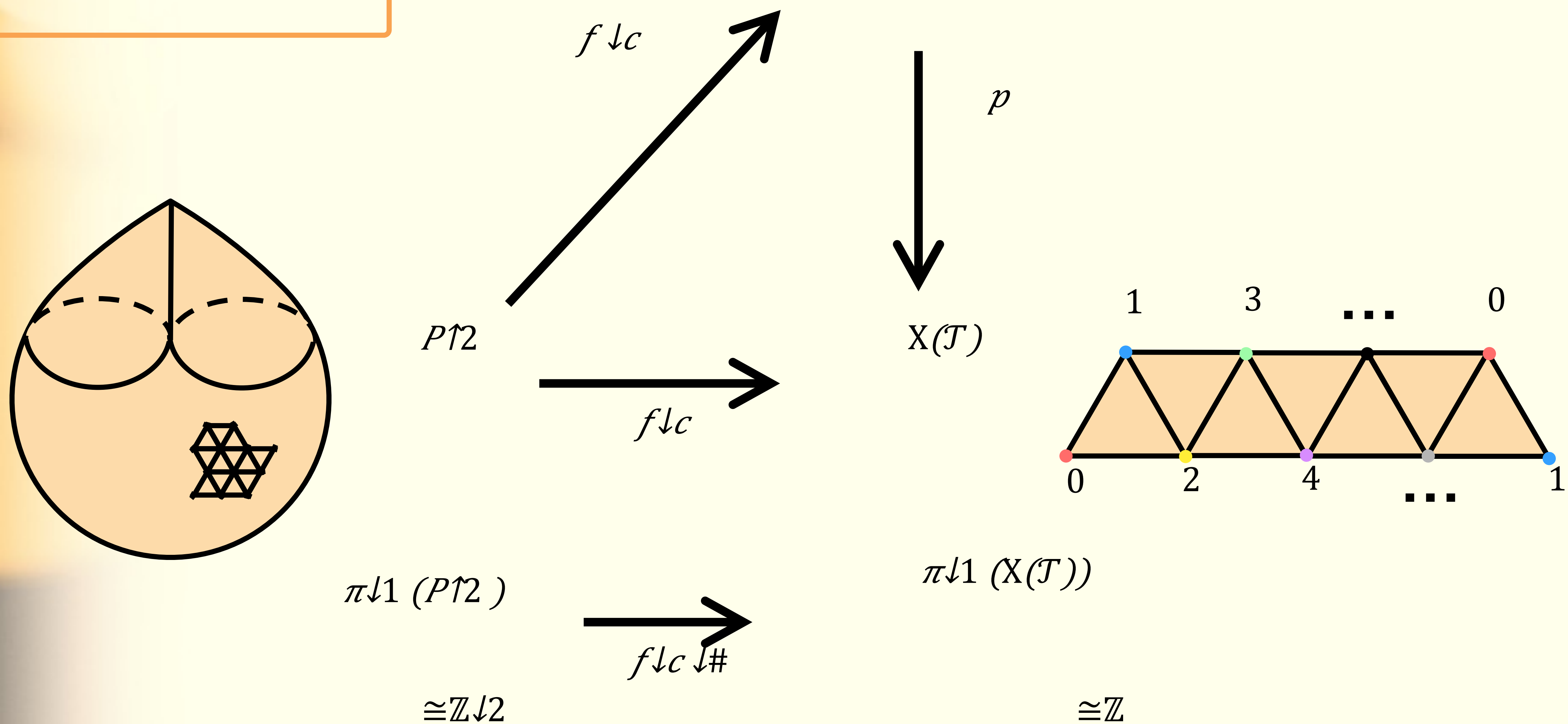
$\cong\{1\}$

$\cong\mathbb{Z}$

$f_{c\#}(\pi_1(P^2))=\{1\}$

$\chi(G)=3$



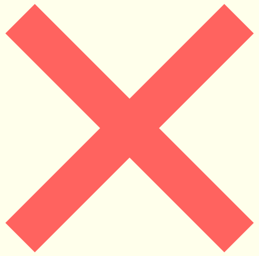


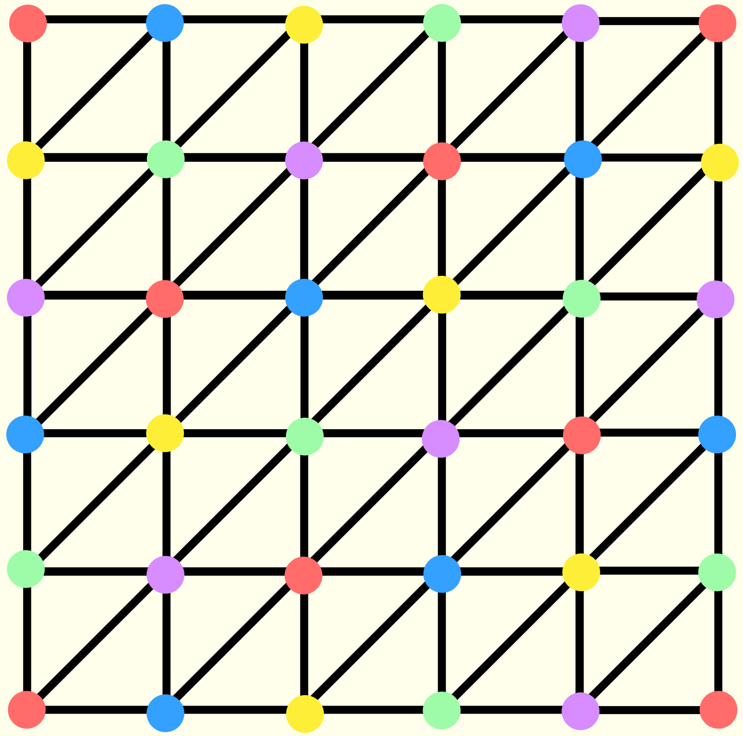
Main theorem

A triangulation G on the sphere or the projective plane is n-triad colorable for some $n \geqq 5$ if and only if $\chi(G)=3$.

$\Leftarrow$

$\Rightarrow$

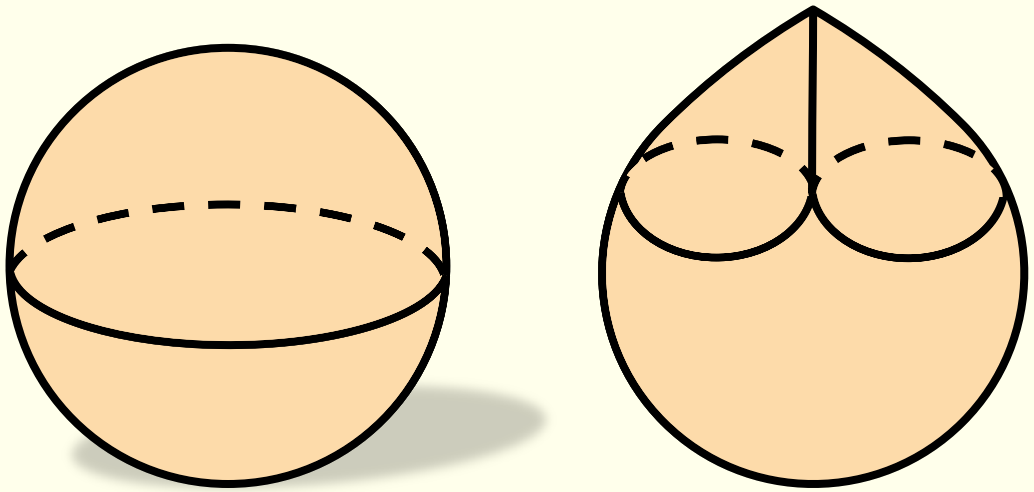




5-triad colorable, $\chi(G)=4$

Remarks

- Any triangulation on the sphere is 4-triad colorable.
- There are many triangulations on the projective plane which are *not* n-triad colorable for any $n \geqq 3$.



Thank you for your attention!!