Extendability of finite group actions on compact surfaces

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Symmetries and Covers of Discrete Objects

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joint work (in progress) with Emilio Bujalance and Marston Conder

Example.

Let S be the compact Riemann surface given by $y^{16} = x(x-1)$.

Let $u : (x, y) \mapsto \left(x, ye^{\pi i/8}\right)$. Then $\langle u \rangle = \mathrm{C}_{16} < \mathrm{Aut}(S)$.

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New Question: Is $\langle u, v \rangle = \mathrm{Aut}(S)$?

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Suppose G is a group of automorphisms of some compact surface S of genus > 1.

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We will consider not only Riemann surfaces but also surfaces which might be non-orientable or with boundary.

The use of algebraic equations is usually very difficult.

In our previous example with $S : y^{16} = x(x-1)$ it turns out that $\langle u, v \rangle = \mathrm{C}_{16} \times \mathrm{C}_2$ is not $\mathrm{Aut}(S)$:

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$$(x, y) \mapsto \left(\frac{1}{2} + \frac{2x-1}{4y^8}, \frac{c^2}{y}\right) \quad \text{where } c^{16} = -1/4,$$

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Instead of algebraic equations, the Uniformization Theorem allows us to use the combinatorial theory of discrete subgroups of isometries of the hyperbolic plane.

Group actions on compact Riemann surfaces

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With this terminology, the problem of extendability can be read as: "When does $\theta : \Gamma \to G$ extend to an epimorphism $\theta' : \Gamma' \to G'$ (with the same kernel Λ) for some larger NEC group Γ' containing Γ?"

$$\begin{array}{ccc} \Gamma' & \xrightarrow{\theta'} & G' \\ \uparrow & & \uparrow \\ \Gamma & \xrightarrow{\theta} & G \end{array}$$

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In *most* cases Γ is not contained as a subgroup of finite index in any other Fuchsian group. If this is the case then the action of the group $G = \Gamma/\Lambda$ cannot be extended.

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Fuchsian pairs $(\sigma(\Gamma), \sigma(\Gamma'))$ with Γ normal in Γ'

$\sigma(\Gamma)$	$\sigma(\Gamma')$	Index
$(2;\ —)$	$(0;\ 2,2,2,2,2,2)$	2
$(1;\ t,t)$	$(0;\ 2,2,2,2,t)$	2
$(1;\ t)$	$(0;\ 2,2,2,2t)$	2
$(0;\ t,t,t,t)$	$(0;\ 2,2,2,t)$	4
$(0;\ t_1,t_1,t_2,t_2)$	$(0;\ 2,2,t_1,t_2)$	2
$(0;\ t,t,t)$	$(0;\ 3,3,t)$	3
$(0;\ t,t,t)$	$(0;\ 2,3,2t)$	6
$(0;\ t_1,t_1,t_2)$	$(0;\ 2,t_1,2t_2)$	2

Fuchsian pairs $(\sigma(\Gamma), \sigma(\Gamma'))$ with Γ not normal in Γ'

$\sigma(\Gamma)$	$\sigma(\Gamma')$	Index
$(0;\ 7,7,7)$	$(0;\ 2,3,7)$	24
$(0;\ 2,7,7)$	$(0;\ 2,3,7)$	9
$(0;\ 3,3,7)$	$(0;\ 2,3,7)$	8
$(0;\ 4,8,8)$	$(0;\ 2,3,8)$	12
$(0;\ 3,8,8)$	$(0;\ 2,3,8)$	10
$(0;\ 9,9,9)$	$(0;\ 2,3,9)$	12
$(0;\ 4,4,5)$	$(0;\ 2,4,5)$	6
$(0;\ t,4t,4t)$	$(0;\ 2,3,4t)$	6
$(0;\ t,2t,2t)$	$(0;\ 2,4,2t)$	4
$(0;\ 3,t,3t)$	$(0;\ 2,3,3t)$	4
$(0;\ 2,t,2t)$	$(0;\ 2,3,2t)$	3

Example: $\sigma(\Gamma) = (0; t, 2t, 2t)$ $\sigma(\Gamma') = (0; 2, 4, 2t)$ index $|\Gamma' : \Gamma| = 4$.

Presentations are

$\Gamma \cong \langle x_1, x_2, x_3 \mid x_1^t = x_2^{2t} = x_3^{2t} = x_1 x_2 x_3 = 1 \rangle$

$\Gamma' \cong \langle y_1, y_2, y_3 \mid y_1^2 = y_2^4 = y_3^{2t} = y_1 y_2 y_3 = 1 \rangle$.

An embedding of Γ in Γ' is given by $x_1 \mapsto y_2 y_3^2 y_2^{-1}$, $x_2 \mapsto y_2^2 y_3 y_2^2$, $x_3 \mapsto y_3$.

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Is $\mathrm{C}_{2t} = \mathrm{Aut}(S)$? We have to determine whether θ can be extended or not.

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\Gamma' & \xrightarrow{\theta'} & G' = ? \\
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- If $\theta(x_2) \neq \theta(x_3),\ \theta(x_3)^{t+1}$ then no extension is possible, so $\mathrm{C}_{2t} = \mathrm{Aut}(S)$.

This approach allows to solve the problem of extendability of group actions when S is a compact Riemann surface:

• G cyclic: [EB & MC]

On cyclic groups of automorphisms of Riemann surfaces,

J. London Math. Soc. 59 (1999), 573–584.

• G arbitrary: [EB, MC & JC]

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We are now working on this question when S is a compact Klein surface.

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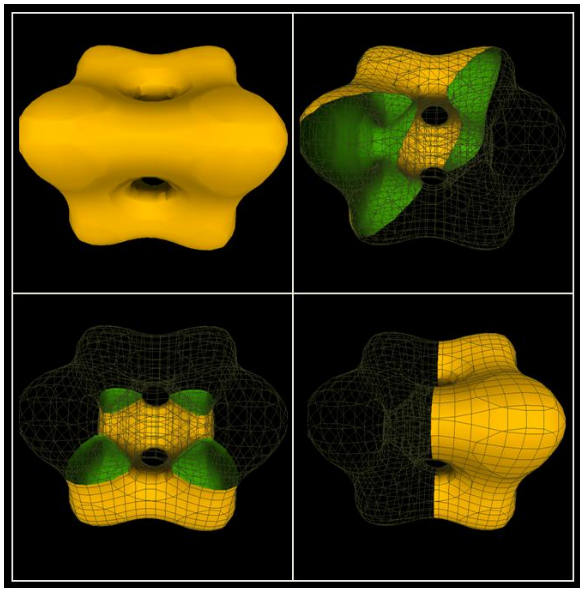

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We also have a list of always non-maximal NEC signatures.

- Normal pairs $(\sigma(\Gamma), \sigma(\Gamma'))$ with $\Gamma \triangleleft \Gamma'$ (Bujalance, 1982),
- Non-normal pairs $(\sigma(\Gamma), \sigma(\Gamma'))$ with $\Gamma \ntriangleleft \Gamma'$ (Estévez & Izquierdo, 2006).

Example.

$\sigma(\Gamma) = (1; -; [t]; \{(-)\})$, $\sigma(\Gamma') = (0; +; [2]; \{(2, 2, t)\})$, index $|\Gamma' : \Gamma| = 2$.

Presentations are $\Gamma \cong \langle d, x, c \mid x^t = c^2 = [d^2 x, c] = 1 \rangle$

$\Gamma' \cong \langle x'_1, c'_0, c'_1, c'_2 \mid (c'_0 c'_1)^2 = (c'_1 c'_2)^2 = (c'_2 x'_1 c'_0 x'_1)^t = 1 \rangle$.

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Assume $S = \mathbb{H}/\Lambda$ is **bordered** and $\theta : \Gamma \to \mathrm{C}_n$ with $\ker\theta = \Lambda$.
QUESTION: Is $\mathrm{C}_n = \mathrm{Aut}(S)$?

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Theorem: (EB, JC, MC, *Rev. Mat. Iberoam.* (2015)):

This happens for all non-maximal NEC signatures! (unlike the Fuchsian case).

Assume $S = \mathbb{H}/\Lambda$ is **unbordered and non-orientable** and $\theta : \Gamma \to \mathrm{C}_n$ with $\ker\theta = \Lambda$.

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Theorem: (EB, JC, MC, *Trans. Amer. Math. Soc.* (2013)):
The action of a cyclic group with non-maximal NEC signature on an unbordered non-orientable surface always extends to the action of a larger group.

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If there exists $\theta : \Gamma \to G$ with $\ker \theta = \Lambda$ then

$$\begin{aligned} \theta : \ \Gamma &\to G \\ d &\mapsto a \\ x &\mapsto b \\ c &\mapsto 1 \end{aligned}$$

So G admits the (partial) presentation $G = \langle a, b \mid b^t = \cdots = 1 \rangle$.

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Consistency with the embedding of Γ in Γ' yields that the action extends if and only if $a \mapsto a^{-1}, \quad b \mapsto b^{-1}$ is an automorphism for the above presentation.

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Similar results are obtained for unbordered non-orientable surfaces.

Thank you!


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